

1) Lafayette Savings Bank offers an account that pays 2% compounded daily. Bank One wants to offer an account that pays the same annual effective rate as Lafayette Bank but is compounded monthly. What interest rate, compounded monthly, should they offer? Sol

$$> i := \frac{.02}{365}$$

$$i := 0.00005479452055 \quad (1)$$

$$> j := 12 \cdot \left((1 + i)^{\frac{365}{12}} - 1 \right)$$

$$j := 0.02001630 \quad (2)$$

2) Purdue P&C Insurance Company expects claims of \$400,000 in 2013, \$300,000 in 2014, and \$150,000 in 2015. Assuming that they can invest funds at 5% interest, how much must Purdue P&C have on reserve on January 1, 2013 in order to be assured of being able to pay all claims? Assume that all claims are paid at the end of the year.

$$> i := .05$$

$$i := 0.05 \quad (3)$$

$$> A := (1 + i)^{-1} \cdot 400000 + (1 + i)^{-2} \cdot 300000 + (1 + i)^{-3} \cdot 150000$$

$$A := 7.826368644 \cdot 10^5 \quad (4)$$

3) You borrow \$B at the beginning of year 1 at 3% annual effective interest. You pay \$1000 at the end of year 1, and \$2,000 at the end of year 2, \$500 at the end of year 3, \$400 at the end of year 4, and \$400 at the end of year 5, after which you owe nothing. Find B.

$$> i := .03; A := (1 + i)^{-1} \cdot 1000 + (1 + i)^{-2} \cdot 2000 + (1 + i)^{-3} \cdot 500 + (1 + i)^{-4} \cdot 400 + (1 + i)^{-5} \cdot 400$$

$$i := 0.03$$

$$A := 4014.074767 \quad (5)$$

4) From January 1, 2000 to December 31, 2004, First Bank paid 5% interest, compounded monthly. On January 1, 2005, they lowered their rate to 3% interest, compounded monthly. I deposited \$100 at the end of each month beginning in January, 2000. How much did I have in my account immediately after my deposit on December 31, 2009

$$> i := \frac{.05}{12}; j := \frac{.03}{12}$$

$$i := 0.004166666667$$

$$j := 0.002500000000 \quad (6)$$

$$> B1 := \frac{((1 + i)^{5 \cdot 12} - 1)}{i} \cdot 100$$

$$B1 := 6800.608895 \quad (7)$$

$$> B := \frac{((1 + j)^{5 \cdot 12} - 1)}{j} \cdot 100 + (1 + j)^{5 \cdot 12} \cdot B1$$

$$B := 14364.37270 \quad (8)$$

5) An account had an annual effective rate of return of 3% over a 5 year period. It earned 3% annual effective discount for the first two years, 3% annual effective force of interest for years 3 and 4, and i % annual effective interest during the 5th year. Find i .

$$> a := .03;$$

$$a := 0.03 \quad (9)$$

$$> i1 := \frac{a}{1 - a}$$

$$i1 := 0.03092783505 \quad (10)$$

$$\begin{aligned} > i2 := \exp(a) - 1 \\ & i2 := 0.030454534 \end{aligned} \quad (11)$$

$$\begin{aligned} > j := (1 + i1)^2 \cdot (1 + i2)^2 \\ & j := 1.128532838 \end{aligned} \quad (12)$$

$$\begin{aligned} > Ans := \frac{(1.03)^5}{j} - 1 \\ & Ans := 0.027240001 \end{aligned} \quad (13)$$

6) You invest \$100 at the beginning of each month in an account that pays 8% interest per year, compounded monthly. How many months will it take for your balance to equal \$1,000,000?

$$\begin{aligned} > i := \frac{.08}{12}; \text{solve}\left((1 + i) \cdot \frac{((1 + i)^x - 1)}{i} \cdot 100 = 1000000, x\right); \\ & i := 0.00666666666667 \\ & 633.3088120 \end{aligned} \quad (14)$$

7) You owe \$50,000 in student loans which you pay with a 20 year loan at 5% interest, compounded monthly, on which you pay \$329.98 at the end of each month. How much do you owe at the end of the fifth year—i.e. immediately after the 60th payment?

$$\begin{aligned} > i := \frac{.05}{12}; \\ & i := 0.00416666666667 \end{aligned} \quad (15)$$

$$\begin{aligned} > (1 + i)^{60} \cdot 50000 - \frac{((1 + i)^{60} - 1)}{i} \cdot 329.98 \\ & 41727.28597 \end{aligned} \quad (16)$$

8) In problem 7, immediately after the 60th payment, you refinance the loan at 3% interest, compounded monthly. Assuming that the answer to Problem 7 is \$30,000 (which is not correct), find the new monthly payment.

$$\begin{aligned} > i := \frac{.03}{12}; n := 15 \cdot 12; \\ & i := 0.002500000000 \\ & n := 180 \end{aligned} \quad (17)$$

$$\begin{aligned} > PV := (1 + i)^{-n} \cdot \frac{((1 + i)^n - 1)}{i} \\ & PV := 144.8054716 \end{aligned} \quad (18)$$

$$\begin{aligned} > \frac{30000}{PV} \\ & 207.1744919 \end{aligned} \quad (19)$$

9) I bought \$80,000 of RC Penney stock on January 1, 2011. I sold \$10,000 worth of RC Penney stock on April 1 and sold \$2000 of RC Penney stock on July 1. On January 1, 2012, I sold all of my RC Penney stock for \$ 71,219.86. Approximate the rate of return on my investment.

$$\begin{aligned} > \text{solve}\left(80000 \cdot (1 + x) - 10000 \cdot \left(1 + \frac{9}{12}x\right) - 2000 \cdot \left(1 + \frac{6}{12}x\right) = 71219.86, x\right) \\ & 0.04503300699 \end{aligned} \quad (20)$$

10) What price should you pay for a \$5,000 face value, 15 year bond which has \$100 coupons, paid two times a year, assuming that you want a 3% yield, compounded twice a year?

$$\begin{aligned} > i := \frac{.03}{2} \\ & i := 0.01500000000 \end{aligned} \quad (21)$$

$$\begin{aligned} > FV := 5000 + \frac{((1+i)^{30} - 1)}{i} \cdot 100 \\ & FV := 8753.868133 \end{aligned} \quad (22)$$

$$\begin{aligned} > PV := (1+i)^{-30} \cdot FV \\ & PV := 5600.395948 \end{aligned} \quad (23)$$

11) The bond in question (10) is sold after two years, immediately after the payment of the coupon, to an investor wanting a 2% yield, compounded quarterly? What should the selling price of the bond be?

$$\begin{aligned} > i := \left(1 + \frac{.02}{4}\right)^2 - 1 \\ & i := 0.010025000 \end{aligned} \quad (24)$$

$$\begin{aligned} > FV := 5000 + \frac{((1+i)^{30-4} - 1)}{i} \cdot 100 \\ & FV := 7953.517746 \end{aligned} \quad (25)$$

$$\begin{aligned} > PV := (1+i)^{-26} \cdot FV \\ & PV := 6136.546692 \end{aligned} \quad (26)$$