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1) Lafayette Savings Bank offers an account that pays a nominal rate of  $i\%$  per year, compounded daily. A deposit of \$1,000 made on January 1, 2008 had grown to \$1,127.49 on January 1, 2013. Find  $i$ .

$$B = 1127.49/1000$$

$$(1+i/365)^{(5 \cdot 365)} = B$$

$$1+i/365 = (B/1000)^{1/(5 \cdot 365)}$$

$$> B := \frac{1127.49}{1000}; i := 365 \cdot \left( B^{\frac{1}{5 \cdot 365}} - 1 \right)$$

$$B := 1.127490000$$

$$i := 0.0239995$$

(1)

2) The value of the Purdue P&C Insurance Company's reserve account was \$850,000 on January 1, 2010. They paid claims of \$400,000 on December 31, 2010, \$300,000 on December 31, 2011, and \$150,000 on December 31, 2012 from the reserve account. Assuming that the reserve account earned 4% interest and they paid all of the claims from this account, what was the value of this account on January 1, 2013?

$$> i := .04; FV := 850000 \cdot (1+i)^3 - 400000 \cdot (1+i)^2 - 300000 \cdot (1+i) - 150000$$

$$i := 0.04$$

$$FV := 61494.4000$$

(2)

3) I just won a prize that promised to pay \$10,000 at the end of the year for  $n$  years, with the first payment made at the end of the current year. I computed that at 4% interest, my prize is worth \$186,646.17 today. Find  $n$ .

$$((1+i)^n - 1)/i \cdot 10000 = 186646.17(1+i)^n$$

$$\text{Let } x = (1+i)^n.$$

$$\frac{(x-1)}{i} = \frac{186646.17}{10000} \cdot x$$

$$\text{Also } \ln(x) = n \cdot \ln(1+i)$$

$$> i := .04; a := \text{solve} \left( x - 1 = i \cdot \left( \frac{186646.17}{10000} \right) \cdot x, x \right); n := \frac{\ln(a)}{\ln(1+i)};$$

$$i := 0.04$$

$$a := 3.946091341$$

$$n := 35.00001516$$

(3)

4) From January 1, 2006 to December 31, 2008, First Bank paid 4% interest, compounded monthly. On January 1, 2009, they lowered their rate to 2% interest, compounded monthly. I deposited \$100 at the end of each month beginning in January, 2006. How much did I have in my account on January 1, 2013?

$$> i := \frac{.04}{12}; j := \frac{.02}{12};$$

$$i := 0.003333333333$$

$$j := 0.001666666667 \quad (4)$$

$$> BI := \frac{((1+i)^{3 \cdot 12} - 1)}{i} \cdot 100$$

$$BI := 3818.155830 \quad (5)$$

$$> B := \frac{((1+j)^{4 \cdot 12} - 1)}{j} \cdot 100 + (1+j)^{4 \cdot 12} \cdot BI$$

$$B := 9128.780348 \quad (6)$$

5) Over a 7 year period, an account earned 7% annual effective interest for the first two years, 7% annual effective force of interest for years 3 through 5, and 7% annual effective discount during the 6th and 7th years. Find the annual effective rate of return on this account.

$$> i := .07; j := \exp(.07) - 1; d := .07; ii := \frac{d}{1-d};$$

$$i := 0.07$$

$$j := 0.072508181$$

$$d := 0.07$$

$$ii := 0.07526881720 \quad (7)$$

$$> k := ((1+i)^2 \cdot (j+1)^3 \cdot (1+ii)^2)^{\frac{1}{7}} - 1$$

$$k := 0.072578461 \quad (8)$$

7) You borrow \$1,000,000 to buy a house which you finance with a 30 year loan at 5% interest, compounded monthly, on which you pay \$5368.22 at the end of each month. How much do you owe at the end of the tenth year—i.e. immediately after the 120th payment?

$$> i := \frac{.05}{12}; n := 10 \cdot 12; M := 5368.22; B := (1+i)^n \cdot 1000000 - \frac{((1+i)^n - 1)}{i} \cdot M;$$

$$i := 0.004166666667$$

$$n := 120$$

$$M := 5368.22$$

$$B := 8.134200408 \cdot 10^5 \quad (9)$$

8) In problem 7, immediately after the 120th payment, you refinance the loan at 4% interest, compounded monthly. Assuming that the answer to Problem 7 is \$800,000 (which is not correct), find the new monthly payment.

$$> i := \frac{.04}{12}; n := 20 \cdot 12; M := \text{solve}\left(\frac{((1+i)^n - 1)}{i} \cdot P = (1+i)^n \cdot 800000, P\right)$$

$$i := 0.003333333333$$

$$n := 240$$

$$M := 4847.842950 \quad (10)$$

9) At the end of year 1 I deposit \$1000 into an account that is earning 4% interest compounded annually. At the end of each subsequent year I deposit 3% less than I did the previous year. Find the final accumulation in the account at the end of year 30.

$$j=1.04; k=.97$$

$$A=1000(j^{29}+k*j^{28}+j^{28}*k^2+\dots+1*k^{29})=1000*j^{29}(1+(k/j)^{28}+(k/j)^{27}+\dots+1)$$

$$=1000j^{29}((k/j)^{30}-1)/(k/j-1) \text{ or}$$

$$A=1000(j^{29}+k*j^{28}+j^{28}*k^2+\dots+1*k^{29})=1000*k^{29}((j/k)^{29}+(j/k)^{28}+\dots+1)$$

$$=1000k^{29}((j/k)^{30}-1)/(j/k-1)$$

$$> j := 1.04; k := .97; \frac{1000 \cdot k^{29} \cdot \left( \left( \frac{j}{k} \right)^{30} - 1 \right)}{\frac{j}{k} - 1}; \frac{1000 \cdot j^{29} \cdot \left( \left( \frac{k}{j} \right)^{30} - 1 \right)}{\frac{k}{j} - 1};$$

$$j := 1.04$$

$$k := 0.97$$

$$40605.57799$$

$$40605.57774$$

(11)

10) What price should you pay for a 15 year bond having a \$5,000 redemption value which has \$200 coupons, paid 4 times a year, assuming that you want a 5% yield, compounded 4 times a year? Note: The 5% is a nominal rate, not an annual effective rate.

$$> i := \frac{.05}{4}; C := 200; n := 15;$$

$$i := 0.0125000000$$

$$C := 200$$

$$n := 15$$

(12)

$$> FV := \frac{((1+i)^{n \cdot 4} - 1)}{i} \cdot 200 + 5000; PV := (1+i)^{-n \cdot 4} \cdot FV;$$

$$FV := 22714.90155$$

$$PV := 10779.75637$$

(13)

11) The bond in the previous question is sold after 5 years, immediately after the payment of the coupon, to an investor wanting a 7% yield, compounded 4 times a year?? What should the selling price of the bond be?

$$> i := \frac{.07}{4}; C := 200; n := 10;$$

$$i := 0.0175000000$$

$$C := 200$$

$$n := 10$$

(14)

$$> FV := \frac{((1+i)^{n \cdot 4} - 1)}{i} \cdot 200 + 5000; PV := (1+i)^{-n \cdot 4} \cdot FV;$$

$$FV := 16446.82678$$

$$PV := 8216.850825$$

(15)

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