CHAPTER ONE

WHY INSURANCE?

1.1 THE EVOLUTION OF INSURANCE

Humans have strived for security since the beginning of their existence. At its earliest point, security existed if there was an assurance of food, warmth, and shelter. The Bible relates the story of how, in ancient Egypt, Joseph set aside part of the crop in good years in an attempt to cover the expected shortfall in years of drought.

As society developed and the roles of individuals within the economic framework became more specialized, the need for economic security increased.

Economic security is the opposite of ***economic risk*** which we will refer to simply as ***risk***. Risk derives from variation from the expected, not from probability. For example, on a cloudy morning we may be told there is a risk of rain. What is meant, more correctly, is that there is a high probability of rain. The variation associated with the weather forecast could be just as high or higher on a sunny morning.

A modern industrial society provides many examples of risk. A homeowner faces a large variation associated with the potential economic loss caused by a house fire. A driver faces a similar, though less variable, potential economic loss if his or her car is damaged. A larger possible economic loss would be associated with the injury of a third party in a car accident for which you are responsible.

Examples of early informal insurance arrangements can be found in the cooperatives and fraternals that existed in Europe over 400 years ago. For example, the farmers in a certain area would agree, usually informally, that if one farmer's barn was destroyed, the community would see that it was rebuilt. If the breadwinner in a family unit died, the community would "pass the hat" to establish a fund for the surviving dependents. In this informal arrangement, each person's economic risk was shared or pooled among the members of the community.

These informal systems proved to be adequate for several hundred years. At the time of the industrial revolution, however, the need for a more formal system arose. Because of the rapid urbanization of the population, it became true that one's neighbor could be a stranger with whom one had no interests in common. Hence, it was no longer sufficient to expect a communal or cooperative response when one family unit met with an economic reversal.

It was perfectly natural that the "pooling" concept of the existing cooperatives and fraternals became formalized in the new insurance industry. Under the new formal arrangement, each policyholder still implicitly pooled his or her risk with all other policyholders. However, it was no longer necessary for any individual policyholder to know or have any connection with any other policyholder.

1.2 HOW INSURANCE WORKS

If we look at the risk profile of an individual, we see that there is an extremely large variation of possible outcomes, each with a specific economic consequence. Thus, any individual is exposed to a significant amount of risk associated with perils like death, fire, disability, and so on.

By purchasing an insurance policy, an individual (the ***insured***) can transfer this risk, or variability of possible outcomes, to an insurance company (the ***insurer***) in exchange for a set payment (the ***premium***). We might conclude, therefore, that if an insurer sells n policies to n individuals, it assumes the total risk of the n individuals. In fact, the insurer, through careful underwriting and selection, ends up with a total risk that is extremely small. In fact, it may be smaller in total than that associated with any one of the individual policyholders.

The explanation of this surprising result is a principle called ***the law of large numbers***, which states that as the number of observations increases, the difference between the observed relative frequency of an event and the true underlying probability tends to zero. Similarly, the difference between the observed average severity of an event (the average size of a loss) and the expected average severity tends to zero as the number of observations increases.

Here is another way to see the reduced variability of outcomes based on larger samples. At a certain age, the probability of death within one year is .0010, or 10 in 10,000. If we have a sample of 10,000 lives, we can predict with 95% probability that the number of deaths will be between 4 and 16, a range of ± 6 away from the mean of 10. If we have a sample of 1,000,000 lives, the 95% confidence interval is (938, 1062), a range of ± 62 away from the mean of 1000. But we observe that the variability is 60% of the mean in the first case, but only 6.2% of the mean in the case with the larger sample.

As long as the individuals being insured are independent risks (*i.e.*, a claim from one policyholder does not increase the probability of a claim from any other policyholder), then the larger the sample size, the smaller the variance of the average claim, and, hence, the smaller the risk. Thus, through the insurance mechanism, individuals can transfer their risks to an insurer without having the insurer taking on an unmanageable level of risk in total.

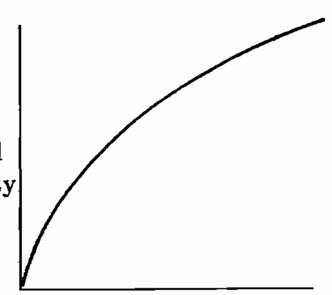
In life insurance, the risk is associated with the variability in the number of death claims, which is modeled by a probability frequency distribution. In most property/casualty lines of insurance (*e.g.*, auto), not only is there a frequency distribution for number of claims, but there is also a severity (or loss) distribution for size of claim, from which variability also arises. That is, given that a claim has occurred, the size of the loss payment is still highly variable.

1.3 INSURANCE AND UTILITY

It should be clear that the existence of a private insurance industry, of and by itself, will not decrease claims frequencies or loss severities. Viewed another way, merely by entering an insurance contract a person's expectation of loss does not change. Thus, with perfect information, the net premium for any policyholder would have to be the expected value of loss. But the policyholder would have to pay a gross premium in excess of the net premium so as to cover the expenses of selling and servicing the contract.

Why would someone pay a gross premium for an insurance contract which must exceed the expected value of the loss? The answer lies in a principle called the ***decreasing marginal utility of money***. According to this principle, as extra units of wealth or income are added, the utility derived from such units decreases. This is displayed in the graphs that follow.

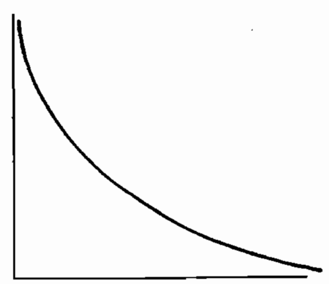
Total Utility



Wealth

Figure 1.1a

Marginal Utility



Wealth

Figure 1.1b

For example, with early dollars of income we buy food, clothing, and shelter, which represent high utility. With later dollars of income, we buy items such as a stereo for the jacuzzi, which is of lower utility.

The principle of decreasing marginal utility of money applies to anyone who is a ***risk avoider***, which is the case for most people. There are some people who are ***risk seekers***, for whom the principle of decreasing marginal utility does not apply. For example, such a person could be expected to forgo basic needs, such as food or shelter, to gamble on a chance for large wealth. The examples that follow assume that the purchaser of insurance is a risk avoider.

Example 1.1

A prospective purchaser of insurance has 100 units of wealth. He faces a situation whereby he could incur a loss of Y units, where Y is a random loss with a uniform distribution between 0 and 36. This person has a personal utility curve given by $u(x) = \sqrt{x}$. What maximum gross premium would this person be willing to pay for insurance?

Solution

Note that for this individual $u'(x) > 0$, so that u increases with x, and $u''(x) < 0$, so that each additional unit of x brings less than one additional unit of utility, u. Hence this prospective policyholder is a risk avoider, since the law of decreasing marginal utility applies. (A risk seeker would have an increasing marginal utility curve.) Further, noting that the p.d.f. for the random loss is $f(y) = \frac{1}{36}$, we can find

$$\begin{aligned} E[Y] &= \int y \cdot f(y)\,dy \\ &= \int_0^{36} \frac{y}{36}\,dy \\ &= \left.\frac{y^2}{72}\right|_0^{36} \\ &= 18, \end{aligned}$$

so the expected value of the loss is 18. Therefore, the insurer must charge a gross premium in excess of 18 to cover sales commissions and administration costs.

Why would a policyholder pay more than 18 to buy insurance whose expected value is 18? The answer lies in the marginal utility curve for this policyholder illustrated in the following figure.

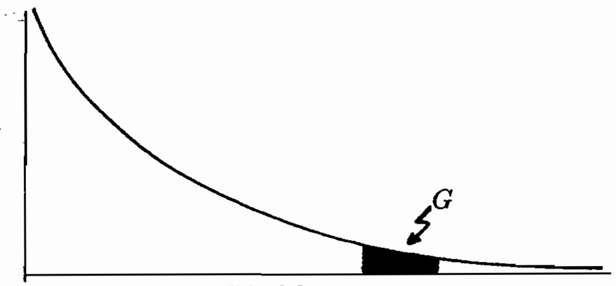


Figure 1.2

The policyholder will pay a gross premium of G for the insurance, so he loses G whether or not the loss occurs, leaving him with $100-G$ units of wealth. Without insurance, however, the policyholder faces a possible loss of 36 units of wealth, which is 36% of his total wealth.

If the policyholder buys insurance, the resulting wealth position is certain; it will be $100-G$, with utility value $\sqrt{100-G}$. If he does not buy insurance, the resulting wealth position is probabilistic, given by $100-Y$, and the expected utility value of the resulting wealth position can be calculated as

$$\begin{aligned} E[U] &= \int_0^{36} u(100-y)\cdot f(y)\,dy \\ &= \int_0^{36} \sqrt{100-y}\cdot\frac{1}{36}\,dy \\ &= \frac{1}{36}\left\{-\frac{2}{3}(100-y)^{3/2}\right\}\Bigg|_0^{36} \\ &= \frac{244}{27}. \end{aligned}$$

The policyholder should be willing to pay a premium G that equates the expected utility values of the resulting wealth positions with or without insurance. Thus we find G such that $\sqrt{100-G}=\frac{244}{27}$, which results in $G=18.33$. Thus the policyholder will pay up to 18.33 for this insurance, which exceeds its expected value of 18, and if the insurer can charge a premium less than 18.33, the insurance purchase will be made. ■

Given this or a similar utility function, we can see why it may not make sense to insure against small losses (*e.g.*, theft of goods worth less than $200). In this case, the utility value of the gross premium will exceed the utility value of the expected loss because we have not moved far enough in the decreasing marginal utility curve to overcome the expense element inherent in the gross premium.

By buying insurance, the individual policyholder transfers his or her risk to the insurer, but, because of the law of large numbers, the insurer ends up with a total risk that is manageable. This is illustrated in Figures 1.3a and 1.3b, showing the risk profiles for the individual and the insurer, respectively.

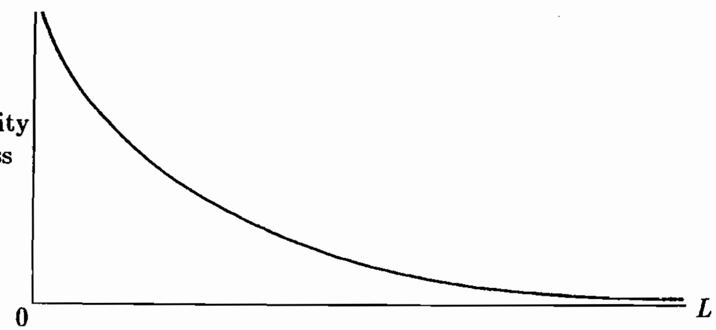


Figure 1.3a

For the individual, the probability is very high that there will be no loss at all from the defined event, but there is a non-zero probability of a significant loss. We denote the expected value of the loss to the policyholder by μ_{ph}, and the variance of the loss to the policyholder by σ^2_{ph}.

If the insurer selects n identical and independent policyholders, each with the same risk profile as that illustrated in Figure 1.3a, then the loss distribution for the insurer can be illustrated by Figure 1.3b.

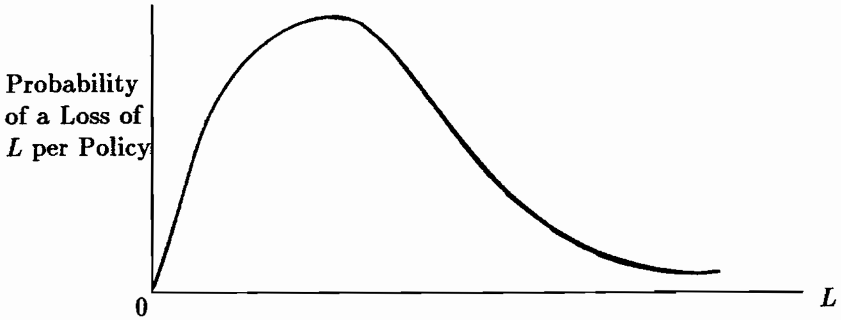


Figure 1.3b

For the insurer, the probability of no loss at all, given n policyholders, will be virtually zero if n is large, and the range of possible losses per policy is much smaller than for the individual policyholder.

If the insurer selects n identical and independent policyholders, the expected value of the average loss per policy is μ_{ph}, the same as for the individual policyholder, but the variance of the average loss per policy is

$$\frac{\sigma^2_{ph}}{n},$$

or, equivalently, a standard deviation of

$$\frac{\sigma_{ph}}{\sqrt{n}}.$$

These results are derived in the following example.

Example 1.2

Given n independent policyholders with individual loss random variables $X_1, X_2, \ldots, X_n$, such that the expected value of any policyholder's loss is μ_{ph} and the variance is σ^2_{ph}, show that for the insurer providing these n policyholders with insurance, the expected value of the insurer's average loss per policy is μ_{ph} and the variance of the average loss per policy is $\frac{\sigma^2_{ph}}{n}$.

Solution

Let $S_n = X_1 + X_2 + \cdots + X_n$.

Let

$$\overline{X} = \frac{1}{n} \cdot S_n = \frac{1}{n}(X_1+X_2+\cdots+X_n).$$

Then

$$E[\overline{X}] = \frac{1}{n} \cdot E[S_n] = \frac{1}{n} \cdot n\mu_{ph} = \mu_{ph},$$

and

$$Var(S_n) = Var(X_1+X_2+\cdots+X_n)$$
$$= n \cdot \sigma^2_{ph}.$$

But

$$Var(\overline{X}) = Var\left(\frac{1}{n} \cdot S_n\right) = \frac{1}{n^2} \cdot Var(S_n)$$
$$= \frac{1}{n^2} \cdot n\sigma^2_{ph}$$
$$= \frac{\sigma^2_{ph}}{n}.$$

Hence we can see that the risk to the insurer (measured by the variance of the average loss) is only $\frac{1}{n}^{th}$ of the risk to the individual policyholder. ■

Example 1.3

You are trying to decide whether to invest in Company A or B. Your personal utility profile can be measured by the function

$$u(P) = \sqrt{P-100}, \quad P > 100,$$

where P represents profit.

(a) Show that this is the utility function of a risk avoider.
(b) Given the following information, determine your investment strategy based on (i) expected monetary value, and (ii) expected utility value.

	Probability	Profit: Company A	Profit: Company B
Economy Advances	.40	4000	2800
Economy Stagnates	.60	200	400

Solution

(a) Given that

$$u(P) = \sqrt{P-100},$$

then

$$u'(P) = \tfrac{1}{2}(P-100)^{-1/2}$$

and

$$u''(P) = -\tfrac{1}{4}(P-100)^{-3/2}.$$

This shows that

$$u'(P) > 0, \quad \text{for} \quad P > 100,$$

and

$$u''(P) < 0, \quad \text{for} \quad P > 100,$$

so the investor is risk averse.

(b) The following table shows the monetary payoffs and their associated utilities.

	Probability	Profit: Company A	Profit: Company B
Economy Advances	.40	4000(62.45)	2800(51.96)
Economy Stagnates	.60	200(10.00)	400(17.32)

(i) Expected monetary value:

$$E(\text{Company A}) = .40(4000) + .60(200) = 1720$$
$$E(\text{Company B}) = .40(2800) + .60(400) = 1360$$

Invest in Company A.

(ii) Expected utility value:

$$E(\text{Company A}) = .40(62.45) + .60(10.00) = 30.98$$
$$E(\text{Company B}) = .40(51.96) + .60(17.32) = 31.18$$

Invest in Company B. ■

1.4 WHAT MAKES A RISK INSURABLE

We have shown in the previous sections that an individual will see the purchase of insurance as economically advantageous if the principle of decreasing marginal utility applies (*i.e.*, the individual is a risk avoider). On the other hand, the insurer will agree to insure a prospective policyholder if the law of large numbers can be applied to the risk pool to which the prospective policyholder wishes to belong. With these principles in mind, what makes a risk insurable?

(1) ***It should be economically feasible.*** If we do not move far enough on the utility function, then the utility gained by insuring will not be enough to cover the

utility of the cost of the insurance mechanism (*e.g.*, sales commissions and head office expenses).

(2) ***The economic value of the insurance should be calculable.*** An example of where this criterion holds is auto collision insurance. Here a large number of small losses are experienced. We can get a lot of data on collision experience and, through the law of large numbers, can calculate an expected premium with a high degree of confidence. Insuring a nuclear reactor against meltdown is an example of where this criterion does not hold. Such a policy can be issued by using a risk-sharing arrangement among many insurers so that the exposure to risk for any one company is manageable.

(3) ***The loss must be definite.*** This criterion is meant to control policyholder manipulation and ***anti-selection.*** Anti-selection occurs when the insured, through concealment of some fact or knowledge, is able to bring a risk to the insurance pool whose cost is expected to exceed the price or premium. A car accident with police documentation is definite. Death is definite. What is not so definite, but still insured, is disability. When is an insured well enough to return to work? How do you guard against malingering?

(4) ***The loss must be accidental in nature.*** Again we wish to have the insured event beyond the control of the policyholder. The presence of criteria three and four allow the actuary to assume random sampling in the projections of future claim activity. That is, there is no statistical bias in the selection of one insurance unit versus another.

(5) ***The exposures in any rate class must be homogeneous.*** This means that, before the fact, the loss expectation for any unit in a class must be the same as for any other unit in the class. In terms of random sampling, this is analogous to each elementary unit having the same probability of being drawn.

(6) ***Exposure units should be spatially and temporally independent.*** In terms of random sampling, this implies that selection of one elementary unit does not affect the probability of drawing any other elementary unit. In more practical terms, we wish to avoid any catastrophic exposure to risk. For example, we would not insure all the stores in one retail area, since one fire or one riot could result in a huge loss. In insurance terms, the fact that one insured has a claim should not affect whether another insured has a claim.

These criteria, if fully satisfied, mean that the risk is definitely insurable. The questions of risk classification and price still follow. On the other hand, the fact that a potential risk exposure does not fully satisfy the criteria does not necessarily mean that insurance will not be issued. However, some special care or risk sharing (*e.g.*, reinsurance) may be necessary. In property/casualty insurance, rarely does an insurable risk meet all of the listed criteria.

1.5 WHAT INSURANCE IS AND IS NOT

There is often confusion in the minds of consumers and regulators as to the purposes and intent of insurance.

The insurance mechanism is used to transfer risk from the individual policyholder to the pooled group of policyholders represented by the insurance corporation. The insurance company administers the plan, invests all funds, pays all benefits, and so on. However, the insurance company can only pay out money that comes from the pooled funds. If claims rise, so too must premiums.

From the policyholders' viewpoint, ***insurance*** is available only for pure risks, a situation where the outcome is either loss or no loss. The policyholder cannot profit from buying insurance.

In ***speculation***, there is also a transfer of risk, in that an individual can transfer an unwanted risk to a speculator. The motive for the speculator is the chance to make a profit.

A good example of how speculation can be used to transfer risk is the futures market. Suppose a farmer has a crop of grain ready for sale in November. If all farmers sold this grain in November the price would fluctuate widely over the year. However, this farmer is risk averse and does not wish to speculate on what the price for grain might be in March. The farmer goes to the futures exchange and sees that it is possible to sell the grain in November to a speculator for $4 a bushel with delivery in March. In March, grain is actually selling for $3.50 a bushel. The farmer delivers the grain as agreed. The speculator must now realize the loss of $.50 a bushel. Had grain risen to $4.50 a bushel, the speculator would have made a profit of $.50 a bushel.

By taking on this risk, the speculator does two positive things. First the risk of fluctuating prices is removed from the farmer who is risk averse. Second, to the extent that speculators are accurate in their forecasts, they provide society with a more level supply of goods, and hence a more level price.

The key difference between speculation and insurance is the profit motive. There is no profit motive on the part of the policyholder in entering an insurance agreement (however, the insurer hopes to make a profit).

In ***gambling***, risk is created where none existed and none needed to exist. In terms of utility, gambling works in a fashion opposite of insurance. People spend early and high utility dollars in the hopes of gaining large wealth that has lower utility value. Overall, gambling decreases societal utility by redistributing income in a non-optimal fashion. Some theorize that gamblers have utility curves that explain their actions, *i.e.*, both $u'(x)$ and $u''(x)$ would be positive.

If the profits from the gambling process (*e.g.*, a state or provincial lottery) are spent on high utility needs (*e.g.*, a hospital), then it is possible for the final result of this process to increase total societal utility. Otherwise gambling decreases total utility and is a waste of human resources.

1.6 RISK, PERIL, AND HAZARD

Risk is a measure of possible variation of economic outcomes. It is measured by the variation between the actual outcome and the expected outcome.

Peril is used as an identifier of a cause of risk. Examples include fire, collision, theft, earthquake, wind, illness, and so on.

The various contributing factors to the peril are called ***hazards***. There are physical hazards such as location, structure, and poor wiring, and there are moral hazards such as dishonesty, negligence, carelessness, indifference, and so on.

An example might help. Mr. Rich owns a cabin cruiser. Hazards when sailing are negligence on the part of the captain, rocks, shoals, and so on. These are contributing factors. Perils would be things like fire or collision (*i.e.*, cause of risk) which may or may not cause a financial loss, which is risk.

In conclusion, an insurance contract will reimburse the policyholder for economic loss caused by a peril covered in the policy. Thus the policyholder transfers this risk to the insurance company.

1.7 EXERCISES

1.1 (a) State the law of large numbers.
(b) Explain the importance of the law of large numbers to the insurance mechanism.

1.2 Confirm that the utility function $u(x) = k \cdot \log x$, for $k > 0$ and $x > 0$, is the utility function of a decision maker who is risk averse.

1.3 Which of the following two proposals would a risk avoider choose?

	Proposal A			Proposal B		
Outcome	Payoff	Utility	Probability	Payoff	Utility	Probability
O_1	80,000	1.0	.6	50,000	.9	.5
O_2	10,000	0.5	.1	30,000	.8	.3
O_3	−30,000	0.0	.3	−10,000	.2	.2

1.4 Two businessmen view the following proposals.

	X		Y	
	Success	Failure	Success	Failure
Profit	50,000	– 20,000	5,000	– 5,000
Probability	.35	.65	.55	.45

Their respective utility values are as follows.

	Businessman	
x	A	B
– 20,000	.300	.550
– 5,000	.450	.709
+ 5,000	.550	.770
+ 50,000	1.000	1.000

What decisions would they make

(a) based on expected monetary value, and
(b) based on expected utility value?

1.5 Assume the management of a firm has utility function $U(P) = \sqrt{P - 1000}$, where P represents profit.

(a) Confirm that management is risk averse.
(b) Consider the following two proposals.

Proposal A		Proposal B	
Profit	Probability	Profit	Probability
3000	.10	2000	.10
3500	.20	3000	.25
4000	.40	4000	.30
4500	.20	5000	.25
5000	.10	6000	.10

Which proposal would management choose

(i) based on expected monetary value, and
(ii) based on expected utility value?

1.6 A market gardener faces the possibility of an early frost that would destroy part of his crop. He can buy crop insurance. This creates four possible outcomes which are presented in the following table.

	Profit	
	Freeze	No Freeze
No Insurance	10,000	30,000
Insurance	20,000	25,000

(a) Based on expected monetary value, what probability must the farmer attach to early frost to make buying insurance a wise decision?
(b) The farmer has the following utility profile.

Profit	Utility
10,000	71
20,000	123
25,000	141
30,000	158

Based on expected utility value, what probability must the farmer attach to an early frost to make buying insurance a wise decision?

1.7 You are subject to the utility function $u(x) = (\frac{x}{10,000})^{.9}$, where x is wealth. Your current wealth is 50,000. What is the maximum premium you would pay to insure against a loss which is uniformly distributed between 0 and 30,000?

1.8 You follow the utility function $u(x) = 1 - exp(\frac{-x}{100,000})$, where x is wealth. Your current wealth is 20,000. What is the maximum amount you would pay to take part in a fair coin toss where you have .5 probability of winning 10,000? If you win you do not receive a return of your wager.

1.9 A person has a utility function, over the relevant range, given by $u(x) = 10{,}000x - x^2$, where x is wealth. Her current wealth is 3000. What is the maximum wager she would make in a game where there is a 30% chance of winning 2000 plus the return of her wager?

1.10 It is common for successful race horses to be sold for stud (breeding purposes) at the end of their racing careers. Not all such horses are "successful." Should it be possible to buy insurance to indemnify you for loss if a race horse you buy is not a successful breeder?

1.11 The XYZ Insurance Company has been asked to issue a 2-year term insurance policy on a specially trained dog that is going to star in a movie. If the dog dies in year one, 8000 will be paid at the end of year one. If the dog dies in year two, 5000 will be paid at the end of year two. If the dog lives to the start of year three, no payment is made and the contract ends. The dog is now age x, and the insurance company develops the following data based on known mortality experience of dogs of the given age and breed.

$$\ell_x = 7000$$
$$\ell_{x+1} = 6000$$
$$\ell_{x+2} = 4500$$
$$\ell_{x+3} = 2500$$
$$\ell_{x+4} = 0$$

(a) Is this an insurable risk?
(b) If $i = 10\%$, determine the net single premium for the contract.
(c) Calculate the associated variance.

1.12 From an economic viewpoint, compare and contrast gambling and insurance. Briefly explain why insurance is more acceptable.

1.13 You are given the following information.

(i) The gross premium for insurance is 4500.
(ii) The individual knows he will have 1, 2, or 3 losses with equal probability.
(iii) Each loss will cost 2000.
(iv) $u = \mu + \sigma/6$ measures the loss of utility for the individual, where u is a measure of utility, μ is the expected value of loss, and σ is the standard deviation of loss.

Under these conditions, determine whether the prospective policyholder will buy insurance. Why?

1.14. Mr. Smith has a total wealth of 525,000 and his utility of wealth is $u(x) = \ln(x)$. He owns a sports car worth 50,000. The insurance on his sports car is due for renewal. Based on Mr. Smith's driving record, the risk of damage to his car in the next year is as follows.

Amount of Damage	Probability
0	.80
10,000	.15
20,000	.04
50,000	.01

Mr. Smith's insurance company charges premiums for all its policies equal to the expected value of its claim payments under the policy plus 10% of this expected value as a loading.

(a) Should Mr. Smith fully insure his car at the insurance company's premium? Explain why or why not.
(b) As an alternative to its full coverage policy, the insurance company is offering a new policy which will pay 50% of all damage amounts for accidents greater than or equal to 20,000. All other damage amounts are paid by the insured. Should Mr. Smith insure his car with this new policy?