STAT 472 Final 2009

Instructions: When the problem says "Express ... in terms of one or more integrals." the answer will be either a single integral or a sum of several integrals. **You need not evaluate the integrals in question.** However, they must be such that if evaluated they would yield a number, not a formula. For example, $\int_0^{20} {}_tp_{50}\, dt$ is not sufficient answer, but $\int_0^{20} \frac{20-t}{20} dt$ is.

Do not put work on the test sheet. Put work on the provided paper, clearly noting the problem number.

1. We issue a fully discrete \$2000 whole life policy to (30) with interest rate $i = .06$, ILT mortality, and the benefit premium. The actuaries computing the reserves at age 40 decide that the original interest rate and mortality assumptions are no longer applicable. They decide that future deaths will be uniformly distributed over the interval $[40, 100]$ and that i should be adjusted down to $i = .05$. Under the revised mortality-interest rate assumptions the policy reserve at age 40 is *8 pts.*

   $${}_{10}V = \qquad\qquad\qquad\qquad .$$

2. Given ${}_{10}V(A_{20}) = .02$, ${}_{10}V(A_{30}) = .07$, then *6 pts.*

   $${}_{20}V(A_{20}) = \qquad\qquad\qquad\qquad .$$

3. We compute that for a totally continuous whole life policy issued to (20) with benefit \$2000 and annual premium \$10 at force of interest $\delta = .05$,

   $$Var({}_{10}L(T(20)) \mid T(20) \geq 10) = 5,000,000.$$

   Let L^* be the loss function for a policy that is identical with the first except that from age 30 on, the benefit is \$2100. Then *8 pts.*

   $$Var({}_{10}L^*(T(20)) \mid T(20) \geq 10) = \qquad\qquad\qquad\qquad .$$

4. Assume that at interest rate $i = .05$, $\ddot{a}_x = 15$, $\ddot{a}_y = 16$ and $\ddot{a}_{\overline{xy}} = 12$. Then *6 pts.*

   $$A_{xy} =$$

5. Assume ILT data and independent lives. What is the APV of a 10 year totally discrete last survivor term policy sold to $\overline{20:30}$? *Warning:* *10 pts.*

$$A_{\overline{20:30}} \neq A^{1}_{\overline{20:30}:\overline{10|}} + {}_{10}E_{\overline{20:30}} A_{\overline{30:40}}.$$

$$A^{1}_{\overline{20:30}:\overline{10|}} =$$

6. For two independent lives (40) and (50). You are given

   (a) The survival function of (40) follows De Moivre's law with $\omega = 100$.

   (b) The survival function of (50) follows De Moivre's law with $\omega = 120$.

   Calculate the probability that $T(40) + T(50) \geq 80$. *8 pts.*

7. For a triple-decrement model with independent decrements, you are given the following information: *10 pts.*

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
37	0.02	0.01	0.03
38	0.04	0.05	0.07
39	0.06	0.06	0.16
40	0.07	0.09	0.02

   Find

   (a) ${}_{2|}q_{37}^{(2)}$

   (b) $\mu_{38}^{(3)}(.3)$ (Assume uniform distribution of the decrements on the interval $(38, 39)$.)

8. I am currently 64. There are three reasons that I might cease work: retirement (decrement (a)), creeping senility (decrement (b)–already all too evident), and DEATH (decrement (c)). Assume that the corresponding "forces of mortality" are *8 pts.*

   (a) $\mu^{(1)}(t) = 1/(10 - t)$, $0 \leq t \leq 10$.

(b) $\mu^{(2)}(t) = .03$.
(c) $\mu^{(3)}(t) = 4t^3$.

(a) Express the probability P that I cease work due to death between ages 66 and 70 in terms of one or more integrals.

$P =$.

(b) Express $\overline{A}_{64}^{(\tau)}$ in terms of one or more integrals.

$\overline{A}_{64}^{(\tau)} =$.

9. Your age is 29 and you want to buy a totally discrete PYD 4-year term life policy with a benefit of 2,000 payable at the end of year of death and premiums 150. (This is not the benefit premium.) Suppose that $i = 0.04$ and $p_{29} = 0.95$, $p_{30} = 0.93$, $p_{31} = 0.9$, $p_{32} = 0.87$. *10 pts.*

   (a) **Use the recursion formula for the reserves** to find the reserve ${}_kV$ for this policy for $k = 2, 3$.

   (b) **Use the recursion formula for the variance of the prospective loss function** ${}_kL$ for this policy (Hattendorf's Theorem) to find for $k = 2, 3$

$$Var({}_kL(29) \mid L(29) \geq k).$$

   The purpose of this question is to test your knowledge of the recursion formulas. Other techniques will not be accepted.

10. A fully discrete 4-year endowment insurance of 20 is issued to (x). The premium is the benefit premium. You are given: *8 pts.*

   (a) $i = 0.2$.
   (b) The reserve at the end of the second year is 4.
   (c) The reserve at the end of the third year is 9.

   (a) Compute the benefit premium P.

   $P =$

(b) Compute p_{x+2}.

$p_{x+2} =$

11. Assume that male mortality is described by $l_x^m = 100 - x$, $0 \leq x \leq 100$ and female mortality has constant force of interest $\mu^f = .03$. Complete each equality with **explicit** quantities, where the male is (40) and the female (60). *10 pts.*

(a)

$${}_{20}p_{\overline{40:60}} =$$

(b)

$${}_{70}p_{\overline{40:60}} =$$

(c)

$$\mu_{40:60}(t) =$$

(d) Express the expected amount of time τ until both of us are dead in terms of one or more integrals. Let δ denote the force of interest.

$$\tau = \qquad\qquad\qquad\qquad .$$

12. Two lives (x) and (y) are not independent. Assume that ${}_tp_x = e^{-t^2}$, ${}_tp_y = e^{-.05t}$ and

$${}_tp_{xy} = \frac{1}{2}({}_tp_y)^2 + \frac{1}{2}\,{}_tp_x\,{}_tp_y$$

Let the force of interest be $\delta = .07$. Express $\overline{A}_{\overline{xy}}$ in terms of one or more integrals. *8 pts.*

$$\overline{A}_{\overline{xy}} =$$