1.) $P = \frac{2000 A_{30}}{\ddot{a}_{30}} = \frac{2000(.10248)}{15.8561} = 12.926256$

${}_{10}V = BA_{40} - \pi\, \ddot{a}_{40}$

$\bar{A}_{40} = \frac{1 - e^{-\delta(60)}}{\delta(60)} = .32331$ $\quad \delta = \ln(1.05)$

$A_{40} = \frac{\delta}{i}\bar{A}_{40} = \frac{\ln(1.05)}{.05}(.32331) = .31549$

$\ddot{a}_{40} = \frac{1 - A_{40}}{d} = \frac{1 - .31549}{\frac{.05}{1.05}} = 14.37471$

${}_{10}V = 2000(.31549) - 12.926256(14.37471)$

${}_{10}V = 445.17$

2.) ${}_{10}V_{20} = .02$ $\quad {}_{10}V_{30} = .07$

${}_{t}V = \left(1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_{x}}\right)$

${}_{10}V_{20} = .02 = \left(1 - \frac{\ddot{a}_{30}}{\ddot{a}_{20}}\right)$ $\quad {}_{10}V_{30} = .07 = \left(1 - \frac{\ddot{a}_{40}}{\ddot{a}_{30}}\right)$

$\frac{\ddot{a}_{30}}{\ddot{a}_{20}} = 1 - .02 = .98$ $\quad \frac{\ddot{a}_{40}}{\ddot{a}_{30}} = 1 - .07 = .93$

$\frac{\cancel{\ddot{a}_{30}}}{\ddot{a}_{20}} \cdot \frac{\ddot{a}_{40}}{\cancel{\ddot{a}_{30}}} = (.98)(.93) = .9114 = \frac{\ddot{a}_{40}}{\ddot{a}_{20}}$

${}_{20}V(A_{20}) = 1 - \frac{\ddot{a}_{40}}{\ddot{a}_{20}} = 1 - .9114$

${}_{20}V(A_{20}) = .0886$

3.) $Var({}_{10}L \cdots\cdots) = (B+\frac{\pi}{\delta})^2({}^2\bar{A}_{x+10} - (\bar{A}_{x+10})^2) = 5{,}000{,}000$

$(2000+\frac{10}{.05})^2({}^2\bar{A}_{x+10} - (\bar{A}_{x+10})^2) = 5{,}000{,}000$

$({}^2\bar{A}_{x+10} - (\bar{A}_{x+10})^2) = 1.033058$

$Var({}_{10}L^*(T(30)) \mid T(30) \geq 10) = (2100+\frac{10}{.05})^2(1.033058)$

$= 5{,}464{,}876.82$

4.) $\ddot{a}_{xy} = \ddot{a}_x + \ddot{a}_y - \ddot{a}_{\overline{xy}} = 15+16-12 = 19$

$A_{xy} = 1 - d\ddot{a}_{xy} = 1 - (\frac{.05}{1.05})(19) = .095238$

5.) $A^1_{\overline{20:30}:\overline{10|}} = A^1_{20:\overline{10|}} + A^1_{30:\overline{10|}} - A^1_{20:30:\overline{10|}}$

$A^1_{20:30:\overline{10|}} = A_{20:30} - {}_{10}E_{20:30} \cdot A_{30:40}$

$= .12867 - (\frac{9501381}{9617802})(\frac{9313166}{9501381})(1.06)^{-10}(.19584)$

$A^1_{20:30:\overline{10|}} = .0227778$

$A^1_{20:\overline{10|}} = A_{20} - {}_{10}E_{20} \cdot A_{30} = .06528 - (.55164)(.10248) = .0087479$

$A^1_{30:\overline{10|}} = A_{30} - {}_{10}E_{30} \cdot A_{40} = (.10248) - (.54733)(.16132) = .0141847$

$A^1_{\overline{20:30}:\overline{10|}} = .0141847 + .0087479 - .0227778 = .0001548$

6.) denote (40) as x, denote (50) as y

${}_tp_x = \frac{60-t}{60}$ ${}_tp_y = \frac{70-t}{70}$

$T(40)+T(50) \geq 80$

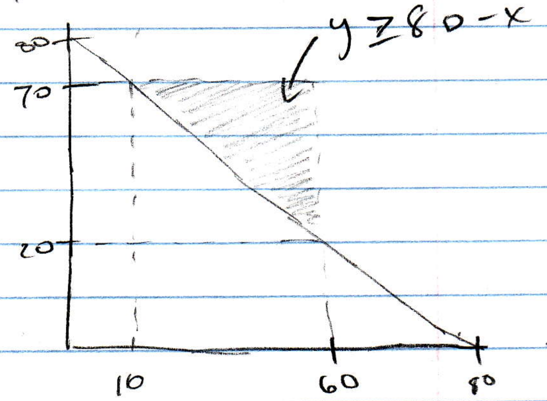
$x+y \geq 80$

$y \geq 80-x$

$Prob(T(40)+T(50) \geq 80)$

$= \frac{\frac{1}{2}(50)(50)}{60 \cdot 70}$

$= .29762$



7. (a) $_{2|}q_{37}^{(2)} = {}_2p_{37}^{(T)} \cdot q_{39}^{(2)}$

$= (1-0.06)(1-0.16)\cdot 0.06$

$= 0.047376$

(b) $\mu_{38}^{(3)}(t) = \dfrac{-\frac{d}{dt}\,{}_tp_{38}^{(3)}}{{}_tp_{38}^{(T)}}$

${}_tp_{38}^{(3)} = 1 - q_{38}^{(3)}\cdot t = 1-0.07t$

$-\frac{d}{dt}\,{}_tp_{38}^{(3)} = 0.07$

${}_tp_{38}^{(T)} = 1 - q_{38}^{(T)}\cdot t = 1-0.16t$

$= \dfrac{0.07}{1-0.16t}$

$\mu_{38}^{(3)}(0.3) = \dfrac{0.07}{1-0.16\times 0.3} = 0.0735294$

8. (a) $\displaystyle\int_2^6 \left(\frac{10-t}{10}\cdot e^{-0.03t}\cdot e^{-t^4}\right)\cdot(4t^3)\,dt$

$\mu^{(1)}(t) = \dfrac{1}{10-t}$ ${}_tp_x'^{(1)} = \dfrac{10-t}{10}$

$\mu^{(2)}(t) = 0.03$ ${}_tp_x'^{(2)} = e^{-0.03t}$

$\mu^{(3)}(t) = 4t^3$ ${}_tp_x'^{(3)} = e^{-\int 4t^3\,dt} = e^{-t^4}$

(b) $\displaystyle\bar{A}_{64}^{(T)} = \int_0^{10}\left(\frac{10-t}{10}\cdot e^{-0.03t}\cdot e^{-t^4}\right)\left(\frac{1}{10-t}+0.03+4t^3\right)e^{-\delta t}\,dt$

9. $2000 A'_{29:\overline{4|}}$ $P=150$ $i=0.04$

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ 29 & 30 & 31 & 32 & 33 \end{array}$$

(a) $$\frac{({}_3V+150)1.04 - 2000 \cdot 0.13}{0.87} = {}_4V = 0$$

$${}_3V = 100$$

$$\frac{({}_2V+150)1.04 - 2000 \cdot 0.1}{0.9} = {}_3V = 100$$

$${}_2V = 128.8462$$

(b) $$Var({}_3V) = 0.87 \cdot 0.13 \left[\frac{1}{1.04}(2000-0)\right]^2 + \cancel{\frac{1}{1.04^2} \cdot 0.87 \cdot 0}.$$
$$= 418269.2308$$

$$Var({}_2V) = 0.9 \cdot 0.1 \cdot \left[\frac{1}{1.04}(2000-100)\right]^2 + \frac{1}{1.04^2} \cdot 0.9 \cdot 418269.2308$$
$$= 648430.388$$

10.

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array}$$

$${}_4V = 20$$

(a) $${}_3V = \frac{20}{1.2} - P = 9 \rightarrow P = 7.6667$$

(b) $$\frac{({}_2V+P)(1+i) - 20 \cdot q_{x+2}}{1-q_{x+2}} = {}_3V$$

$$(4+7.6667)1.2 - 20 q_{x+2} = 9 - 9 q_{x+2}$$
$$14$$

$$5 = 11 q_{x+2}$$

$$q_{x+2} = \frac{5}{11}$$

$$p_{x+2} = \frac{6}{11} = 0.54545$$

11. (a) $m: \ {}_tp_x = \frac{60-t}{60} \quad \mu_x(t) = \frac{1}{60-t}$

$f: \ {}_tp_x = e^{-0.03t} \quad \mu_x(t) = 0.03$

$${}_{20}p_{\overline{40:60}} = {}_{20}p_{40} + {}_{20}p_{60} - {}_{20}p_{40:60}$$

$$= \frac{40}{60} + e^{-0.03\times 20} - \frac{2}{3}e^{-0.6}$$

$$= \boxed{\frac{2}{3} + e^{-0.6} - \frac{2}{3}e^{-0.6}}$$

(b) $${}_{70}p_{\overline{40:60}} = 0 + e^{-0.03\times 70} - 0$$

$$= \boxed{e^{-2.1}}$$

(c) $$\mu_{40:60}(t) = \mu_{40}(t) + \mu_{60}(t)$$

$$= \boxed{\frac{1}{60-t} + 0.03 \quad \text{when } 0 \leq t < 60}$$

(d) $$\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}$$

$$= \boxed{\int_0^{60} \frac{60-t}{60}\,dt + \int_0^{\infty} e^{-0.03t}\,dt - \int_0^{60} \frac{60-t}{60} e^{-0.03t}\,dt}$$

$$= \frac{60t - \frac{1}{2}t^2}{60}\Big]_{t=0}^{t=60} - \frac{100}{3}(0-1) - \frac{1}{60}\int_0^{60}(-t)e^{-0.03t}\,dt$$

$$60\,\mathring{e}_{xy} = \int_0^{60} e^{-0.03t}\,dt - \int_0^{60} t\,e^{-0.03t}\,dt$$

$$= -\frac{100}{3}(e^{-0.03\times 60} - 1) - \left(\frac{100}{3}\right)^2(-0.03t-1)\,e^{-0.03t}\Big]_0^{60}$$

$$=$$

12. $\mu_x = 2t \qquad \mu_y = 0.05$

$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy}$$

$$\bar{A}_x = \int_0^\infty e^{-t^2} 2t\, e^{-0.07t}\, dt = \int_0^\infty 2t\, e^{-t^2-0.07t}\, dt$$

$$\bar{A}_y = \int_0^\infty e^{-0.05t}\, 0.05\, e^{-0.07t}\, dt = \int_0^\infty 0.05\, e^{-0.12t}\, dt$$

$$ {}_tP_{xy} = \frac{1}{2}\left(e^{-0.05t}\right)^2 + \frac{1}{2} e^{-t^2} e^{-0.05t}$$
$$= \frac{1}{2} e^{-0.1t} + \frac{1}{2} e^{-t^2-0.05t}$$

$$\therefore \cancel{\bar{A}_{xy} = \int_0^\infty \left(\frac{1}{2} e^{-0.1t} + \frac{1}{2} e^{-t^2-0.05t}\right)(2t+0.05)\, e^{-0.07t}\, dt}$$

Answer:

$$\boxed{\begin{aligned}\therefore \bar{A}_{\overline{xy}} &= \int_0^\infty 2t\, e^{-t^2-0.07t}\, dt \\ &+ \int_0^\infty 0.05\, e^{-0.12t}\, dt \\ &- \int_0^\infty \left(\frac{1}{2} e^{-0.1t} + \frac{1}{2} e^{-t^2-0.05t}\right) {}_t\mu_{xy}\, e^{-0.07t}\, dt\end{aligned}}$$

$${}_t\mu_{xy} = \frac{-\frac{d}{dt}\, {}_tP_{xy}}{{}_tP_{xy}}$$

$$= -\frac{-0.05\, e^{-0.1t} - \frac{1}{2}(2t+0.05)\, e^{-t^2-0.05t}}{\frac{1}{2} e^{-0.1t} + \frac{1}{2} e^{-t^2-0.05t}}$$