A Field Guide to Some formulas

Crucial formulas:
Probabilities:

$$
\begin{aligned}
p_x &= {}_1p_x \\
{}_tp_x &= \Pr[(x) \text{ lives to age } x+t] \\
&= p_x \cdot p_{x+1} \cdot \ldots \cdot p_{x+t} \\
&= \frac{l_{x+t}}{l_x} \\
{}_tq_x &= \Pr[(x) \text{ dies between ages } x \text{ and } x+t] \\
&= 1 - {}_tp_x \\
{}_{t|u}q_x &= \Pr[(x) \text{ dies between ages } x+u \text{ and } x+t+u] \\
&= {}_up_x - {}_{t+u}p_x \\
&= \frac{l_x - l_{x+t}}{l_x} \\
&(\text{``}|u\text{'' defers any effect } u \text{ years}) \\
{}_tp_x q_{x+t} &= \frac{l_{x+t} - l_{x+t+1}}{l_x} \\
&= {}_tp_x - {}_{t+1}p_x \\
&= {}_{1|u}q_x \\
&= {}_{|u}q_x \\
\tau &= \frac{1}{d} \ln\left(\frac{Sd + P - e}{P - e - Id}\right) \\
\Pr(L_0^g \leq 0) &= {}_{[\tau]}p_x,\ [\,\cdot\,] \ = \ \text{greatest integer function}
\end{aligned}
$$

Discrete

$$
\begin{aligned}
{}_tE_x &= {}_tp_x \nu^t \\
A^{1}_{x:\overline{n}|} &= \sum_{t=0}^{n-1} {}_tp_x q_{x+t} \nu^{t+1} \\
A^{1\,(m)}_{x:\overline{n}|} &= \sum_{t=0}^{mn-1} {}_{t/m}p_x \; {}_{1/m}q_{x+t/m} \nu^{t/m+1} \\
A^{(m)}_x &= A^{1\,(m)}_{x:\overline{\infty}|} \\
A_{x:\overline{n}|} &= A^{1}_{x:\overline{n}|} + {}_nE_x \\
\ddot{a}_{x:\overline{n}|} &= \sum_{t=0}^{n-1} {}_tp_x \nu^t \\
\ddot{a}^{(m)}_{x:\overline{n}|} &= \frac{1}{m} \sum_{t=0}^{nm-1} {}_{t/m}p_x \nu^{t/m} \\
\ddot{a}_x &= \sum_{t=0}^{\infty} {}_tp_x \nu^t \\
a_{x:\overline{n}|} &= \sum_{t=1}^{n} {}_tp_x \nu^t \\
&= \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x \\
e_x &= \sum_{k=1}^{\infty} {}_kp_x \\
E(K_x) &= e_x \\
E(K_x^2) &= 2\sum_{k=1}^{\infty} k({}_kp_x) - e_x \\
e_{x:\overline{n}|} &= \sum_{t=1}^{n} {}_tp_x \\
&= a_{x:\overline{n}|} \text{ with } \nu = 1\,(i=0) \\
P &= \frac{A_x}{\ddot{a}_x} \\
P^{(m)} &= \frac{A_x}{\ddot{a}^{(m)}_x}
\end{aligned}
$$

Note: This needs to be divided by m to give the $m$thly payment.

$$
\begin{aligned}
L_0^g &= (S + \frac{P-e}{d})v^{k_x+1} + I - \frac{P-e}{d} \\
I &= \text{ initial expense, } e = \text{ recurring expense}
\end{aligned}
$$

Continuous:

$$
\begin{aligned}
S(x) &= S_0(x) \\
&= {}_xp_0 \\
S_x(t) &= {}_tp_x \\
&= \frac{S(x+t)}{S(x)} \\
f_x(t) &= -\frac{\frac{\partial}{\partial t} l_{x+t}}{l_x} \\
&= -\frac{\partial}{\partial t} S_x(t) \\
&= {}_tp_x \mu_{x+t} \\
\overline{A}^1_{x:\overline{n}|} &= \int_0^n \nu^t \; {}_tp_x \mu_{x+t}\, dt \\
&= \int_0^n \nu^t f_x(t)\, dt \\
\overline{A}_x &= \overline{A}^1_{x:\overline{\infty}|} \\
\overline{A}_{x:\overline{n}|} &= \overline{A}^1_{x:\overline{n}|} + {}_nE_x \\
\mathring{e}_x &= \int_0^\infty {}_tp_x\, dt \\
E(T_x) &= \mathring{e}_x \\
E(T_x^2) &= 2\int_0^\infty t\; {}_tp_x\, dt \\
\overline{a}_{x:\overline{n}|} &= \int_0^n {}_tp_x \nu^t\, dt \\
\overline{a}_x &= \overline{a}_{x:\overline{\infty}|} \\
\mathring{e}_{x:\overline{n}|} &= a_{x:\overline{n}|} \text{ with } \nu = 1\, (i=0)
\end{aligned}
$$

De Moivre, also called "uniform":

$$
\begin{aligned}
l_x &= \omega - x \\
\overline{A}^1_{x:\overline{n}|} &= \frac{1 - e^{-n\delta}}{\delta(\omega - x)} \\
\overline{A}_x &= \overline{A}^1_{x:\overline{\omega - x}|} \\
A^1_{x:\overline{n}|} &= \frac{\delta}{i}\overline{A}^1_{x:\overline{n}|}
\end{aligned}
$$

Exponential, also called "constant force of mortality"

$$
\begin{aligned}
S(x) &= e^{-\mu x} \\
{}_tp_x &= e^{-\mu t} \\
\overline{A}^1_{x:\overline{n}|} &= \frac{\mu}{\mu + \delta}\left(1 - e^{-(\mu+\delta)n}\right) \\
\overline{A}_x &= \frac{\mu}{\mu + \delta} \\
\overline{a}_{x:\overline{n}|} &= \mu^{-1}\overline{A}^1_{x:\overline{n}|} \\
&= \frac{1}{\mu + \delta}\left(1 - e^{-(\mu+\delta)n}\right)
\end{aligned} \tag{1}
$$

Below is a summary of some formulas and their range of validity, when suitably modified. In particular discrete formulas use $d = i/(1+i)$ instead of $\delta = \ln(1+i)$. Things payable $m$thly (m) will use $d^{(m)}$ and $i^{(m)}$ instead of d and i. "C" means "continuous," "D" means "discrete," and "WL" means "whole life".

Table 1: Life Table Formulas

Formula	*Valid*
$l_{x+t} = (1-t)l_x + tl_{x+1}$	UDD, $0 \le t \le 1$
${}_tq_x = t(q_x)$	UDD, $0 \le t \le 1$
$l_{x+t} = (l_x)^{(1-t)} \cdot (l_{x+1})^t$	CFM, $0 \le t \le 1$
${}_tp_x = (p_x)^t$	CFM, $0 \le t \le 1$
$\mu_{x+t} = \frac{q_x}{1-t(q_x)}$	UDD, $0 \le t \le 1$
$a_{x:\overline{n}\rvert} = a_x - {}_nE_x a_{x+n}$	C,D,(m)
$\ddot{a}_{x:\overline{n}\rvert} = \ddot{a}_x - {}_nE_x \ddot{a}_{x+n}$	C,D,(m)
$A^1_{x:\overline{n}\rvert} = A_x - {}_nE_x A_{x+n}$	T, C, D,
$A_x = q_x v + p_x A_{x+1}$	D,(m)
$\overline{A}_x + \delta \overline{a}_x = 1$	C,D,(m),T,
$\overline{A}_x = \frac{i}{\delta} A_x$	UDD, De Moivre, WL, T,(m)
$\overline{A}_x = (1+i)^{\frac{1}{2}} A_x$	Claims Acceleration
$\overline{A}^{(m)}_x = (1+i)^{\frac{m-1}{2m}} A_x$	Claims Acceleration
$Var(\text{“}\overline{A}_x\text{”}) = {}^2\overline{A}_x - (\overline{A}_x)^2$	C, D,WL, T, E, (m)
$Var(\text{“}\overline{a}_x\text{”}) = \frac{1}{\delta^2}\left({}^2\overline{A}_x - (\overline{A}_x)^2\right)$	C, D,WL, T,(m)
$Var(L^g_0) = (S + \frac{P-e}{d})^2({}^2A_x - (A_x)^2)$	C, D,WL, E, (m)
$Var(\text{“}S\overline{A}_x - P\overline{a}_x\text{”}) = \left(S + \frac{P}{\delta}\right)^2 \left({}^2\overline{A}_x - (\overline{A}_x)^2\right)$	C, D,WL
$= S^2 \frac{{}^2\overline{A}_x - (\overline{A}_x)^2}{(1-\overline{A}_x)^2}$	benefit premium, C, D,WL
$E({}_tL(x) \mid L(x) \ge t) = {}_tV$	
${}_tV = SA_{x+t} - P\ddot{a}_{x+t}$	WL
$= S\left(1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}\right)$	benefit premium, C, D,WL,E
$= S\frac{A_{x+t} - A_x}{1-A_x}$	benefit premium, C, D,WL,E
${}_tV = (b_{t+1}q_{x+t}v - P_t) + p_{x+t}v({}_{t+1}V)$	Discrete, T, WL
$Var({}_tL(x) \mid L(x) \ge t) = \left(S + \frac{P}{\delta}\right)^2 \left({}^2\overline{A}_{x+t} - (\overline{A}_{x+t})^2\right)$	C, D,WL
$= S^2 \frac{{}^2\overline{A}_{x+t} - (\overline{A}_{x+t})^2}{(1-\overline{A}_x)^2}$	benefit premium, C, D,WL
$Var({}_kL) = p_{x+k}q_{x+k}[v(b_{k+1} - {}_{k+1}V)]^2$	
$+ v^2 p_{x+k} Var({}_{k+1}L)$	General
$\ddot{a}^{(m)}_x = \alpha(m)\ddot{a}_x - \beta(m)$	UDD, not temp., not $A^{(m)}_x$
$A^{(m)}_x = \frac{i}{i^{(m)}} A_x$	UDD, WL, T, De Moivre, not $\ddot{a}^{(m)}_x$