Notation and Terminology used on Exam MLC

Version: March 1, 2012

In actuarial practice there is notation and terminology that varies by country, by application, and by source. The purpose of this study note is to present notation and terminology that will be used on the MLC examination for situations where notation or terms may not be common to all sources and notation or terms that are unique to the MLC examination.

The **force of mortality** may be represented by μ_x or $\mu(x)$ or μ_{x+t} or $\mu(x+t)$ where x and x+t are attained ages. The symbol $\mu_x(t)$ indicates selection at age x and attained age $x+t$. The symbols $\mu_{x,}$ μ_{x+t}, and $\mu_x(t)$ will be used on the examination.

The **survival function** may be represented by $s(x)$ or $S(x)$ or $S_x(t)$. The symbol $S_x(t)$ will be used on the examination.

The **number of lives** at age x can be represented by ℓ_x or l_x. The symbol l_x will be used on the examination.

The **future complete lifetime of (x) random variable** can be represented by T_x or $T(x)$. Similarly, the symbols used for joint life status can be T_{xy} or $T(xy)$, and for last survivor status can be $T_{\overline{xy}}$ or $T(\overline{xy})$. The symbols T_x, T_{xy} and $T_{\overline{xy}}$ will be used on the examination.

The **future curtate lifetime of (x) random variable** can be represented by K_x or $K(x)$. Either symbol may be used on the exam. . Similarly, the symbols used for joint life status, K_{xy} or $K(xy)$, and last survivor status, $K_{\overline{xy}}$ or $K(\overline{xy})$. The symbols K_x, K_{xy} and $K_{\overline{xy}}$ will be used on the examination.

The **present value of future losses random variable** may be represented by L or ${}_0L$ or L_0 for loss at issue and ${}_tL$ or L_t for loss from t years after issue. Superscripts may be included. When the symbol L is used to represent present value of future losses random variable the symbol including any subscripts or superscripts will be defined in the text of the question.

Duration subscripts can be used differently. For example, something happening in the first duration (between ages x and $x+1$) may be identified with a 0 or 1. The text of the question will define any notation used.

The **benefit** at time t is represented by b_t. The **benefit** at the end of period k is represented by b_k. When a benefit is payable at the beginning or end of a time period, the subscript will denote the time of the payment, not necessarily the number assigned to the time period. The text of the question will define the benefit either by formula or in words.

Actuarial present value and **expected present value** are terms used for the expectation of the present value random variable for a future payment. Either term may be used on the examination. **Actuarial accumulated value** is the term used for the expectation of the accumulated value random variable of a future payment.

Fully discrete insurance is an insurance where both the premiums and the benefits are paid only at discrete time points. **Semi-continuous** insurance is an insurance where the premiums are paid

at discrete time points and the death benefits are paid at the moment of death. **Fully continuous** insurance is an insurance where the premiums are paid continuously and the death benefits are paid at the moment of death. Unless stated otherwise in the text of the question discrete time points are the beginnings of years for premium payments and the ends of years for death benefit payments.

Special insurance is an insurance that has either: non-level benefits or non-level premiums; or both. The non-level aspects of the insurance will be described in the text of the question. If an insurance is not defined as "special" then premiums and benefits are assumed to be level, unless there is explicit information in the text of the question to the contrary.

Benefit premium is the premium determined by the equivalence principle and assuming no expenses.

The **benefit premium for fully discrete insurances** will be represented by *P* with the appropriate symbols attached. P_x , $P_{x:\overline{n}|}^{1}$, $P_{x:\overline{n}|}^{\frac{1}{}}$, and $P_{x:\overline{n}|}$ may be used on the exam.

The symbols are defined in terms of an insurance, *A*, and an annuity, $\ddot{a}$, as follows:

$$P_x = \frac{A_x}{\ddot{a}_x} \ , \ P_{x:\overline{n}|}^{1} = \frac{A_{x:\overline{n}|}^{1}}{\ddot{a}_{x:\overline{n}|}} \ , \ P_{x:\overline{n}|}^{\frac{1}{}} = \frac{A_{x:\overline{n}|}^{\frac{1}{}}}{\ddot{a}_{x:\overline{n}|}} \ , \ P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

The symbol *P* will be defined within the text of the question if it is not one of the symbols shown above.

Benefit reserves are reserves based on the benefit premium determined by the equivalence principle and assuming no expenses.

Unless stated otherwise in the text of a question all **expenses** are equal to zero. If expenses are specified in the text of a question then the expenses need to be used in the solution to the question.

In a **multi-decrement table** $q_x^{(j)}$ is the probability a life age *x* fails in the next year due to decrement (*j*) and $q_x^{(\tau)}$ is the probability of failure due to all decrements. The associated single decrement probability of failure at age *x* due to decrement (*j*) is $q_x'^{(j)}$. The probability $q_x^{(j)}$ is also called the dependent probability. The probability $q_x'^{(j)}$ is also called the independent probability.

In a **multi-state model** ${}_tp_x^{ij}$ is the probability that a life age *x* in state *i* is in state *j* at age *x*+*t*.

The

symbol μ_x^{ij} is the force of transition between states *i* and *j* at age *x*. The symbol ${}_tp_x^{\overline{ii}}$ is the probability that a life age *x* in state *i* remains in state *i* through the period *x* to *x*+*t*.

Notation and Terminology used on Exam MLC

Version: March 1, 2012

The **asset share** at time t may be represented by ${}_tAS$ or AS_t. The symbol ${}_tAS$ will be used on the examination.

The **reserve** at time t may be represented by ${}_tV$ or V_t. The symbol ${}_tV$ will be used on the examination.

If a **table of select and ultimate values** is presented in a question the format of the table will follow the convention of reading across the row of select rates and then down the column of ultimate rates for the values corresponding to each age at selection.

On the examinations the **transition probabilities** for a multistate model may be presented in a matrix. For example, for a model with two states, 0 and 1, the transition probabilities would be presented in a matrix as follows:

$$\begin{vmatrix} p_x^{00} & p_x^{01} \\ p_x^{10} & p_x^{11} \end{vmatrix}$$

For a time period where all cash flows occur only at the beginning and end of the time period:

Profit for the time period occurs at the end of the period and is (a) minus (b) where:

(a) is the accumulated value (using the period effective interest rate) of the sum of the reserve at the beginning of the period and the cash flows that occur at the beginning of the period; and

(b) is the sum of the value of the reserve at the end of the period and the cash flows that occur at the end of the period.

Expected profit is the profit calculated using the values that were anticipated for the next time period prior to the start of that time period.

Actual profit is the profit calculated using the values that were observed during the time period.

Gain is actual profit less expected profit for the same time period. **Gain by source** is the gain calculated where the observed values and the anticipated values in the profit calculations from one source are changed while the others are not. Examples of sources are: expenses, interest, mortality and lapse.

Internal rate of return is the interest rate such that the actuarial present value at issue of profits is equal to 0.

Profit margin is the ratio of the actuarial present value at issue of profits divided by the actuarial present value of premiums.

Notation and Terminology used on Exam MLC

Version: March 1, 2012

Other terms	
Term used on the examination	Equivalent or similar terms (**not** used on the examination)
annuity-due	due annuity
annuity-immediate	immediate annuity
temporary life annuity	term annuity
certain period	guarantee period
premium paying period	premium paying term
face amount	sum assured, sum insured
net amount at risk	death strain at risk, sum at risk, amount at risk
benefit premium	net premium
gross premium	contract premium, expense-loaded premium, expense-augmented premium
benefit reserve	net premium policy value, net premium reserve
gross premium reserve	gross premium policy value
cost of insurance	COI, mortality charge
μ_x^{ij}	$\mu_{ij}(x)$, $\lambda_{ij}(x)$
${}_tp_x^{ij}$	${}_tp_{ij}^{(x)}$
terms used to describe a universal life insurance death benefit	
specified amount	Option A, Type A, fixed failure benefit
specified amount plus the account value	Option B, Type B, variable failure benefit

Notation and Terminology used on Exam MLC
Version: March 1, 2012

Universal Life Account Value Mechanics

While all universal life insurance policies have similar structures, there are variations, especially with respect to cost of insurance calculations. The following structure applies to universal life policies on the exam unless otherwise specified. If variations to this structure occur on the exam the variation will be stated in the applicable question.

Account values are calculated at regular intervals. The question will indicate the calculation period.

For each calculation period the account value is calculated as follows:

$$AV_{end} = [AV_{start} + P(1-f) - e - COI](1+i^c)$$, where $$COI = \frac{DB_{end} - AV_{end}}{1+i^q}(coi\ rate)$$

The symbols in the equations above will not be used on the exam unless the symbols are also defined in the question.

AV_{start} is the account value at the start of the period.

AV_{end} is the account value at the end of the period.

P is the premium paid (unless otherwise stated in the question, all premiums are paid)

f is the percent of premium charge.

e is the expense charge.

DB_{end} is the projected death benefit at the end of the period, consistent with AV_{end}, reflecting the policy's death benefit description and the corridor factor if specified.

COI is the cost of insurance charge.

coi rate is the cost of insurance rate.

i^c is the credited interest rate per period.

i^q is the interest rate per period for discounting the net amount at risk in the COI calculation (unless otherwise stated in the question $i^q = i^c$)

Death benefits and surrender benefits are paid at the end of the period, after the account value at the end of the period has been calculated.

The **cash surrender value** is the account value at the end of the period minus the applicable surrender charge, if any.

Notation and Terminology used on Exam MLC
Version: March 1, 2012

A universal life policy will lapse if the cash surrender value is 0. Except, a universal life policy may remain in force, even if the cash surrender value is below 0, if the policy has a **no-lapse guarantee** provision and satisfies the conditions of the no-lapse guarantee provision. You should assume that a universal life policy does not lapse unless you are told in the question to check if it lapses.

Corridor factors:

a. Universal life policies may contain a provision that the death benefit will be the greater of: (1) the specified amount; and (2) the account value at the end of the period times a corridor factor. For example, if the specified amount is 50,000, the account value is 30,000, and the corridor factor is 200%, then the death benefit would be 60,000 which is the greater of 50,000 and 200% of 30,000.

b. If a corridor factor provision would increase the death benefit, the cost of insurance charge calculation should reflect the increased death benefit in the calculation of the cost of insurance charge.

c. If no corridor factors are stated in the question, assume the policy has none.

d. If corridor factors are stated in the question, you should consider the corridor factors in any calculations.