STAT 473, Test 1, Fall 2008

Important: In all cases, unevaluated expressions, such as $(1 - e^{-(.03)6})/(1 + (1.03)^{-2})$ and $\frac{1}{65}\int_0^5 e^{-.05\,t}\,dt$ are just as acceptable as numeric answers, as long as the answer can in principle be reduced to a number. It is even acceptable to give an answer of the form "$P = A_x/\ddot{a}_x$ where A_x is as computed in problem 1a and $\ddot{a}_x = \ldots$".

(1) For last-survivor whole life insurance of 10,000 on (20) and (30), you are given:
  (a) The death benefit is payable at the moment of the second death.
  (b) The independent random variables $T^*(20)$, $T^*(30)$ and Z are the components of a common shock model.
  (c) $T^*(30)$ has a De Moivre distribution with $l_{30}(t) = 70-t$, $0 \le t \le 70$.
  (d) $T^*(20)$ has a De Moivre distribution with $l_{30}(t) = 80-t$, $0 \le t \le 80$.
  (e) Z, the common shock random variable, has an exponential distribution with force $\mu_z = 0.03$.
  (f) $\delta = 0.05$.

  Give an expression for the APV of this life insurance.

(2) I am currently 62. There are three reasons that I might cease work: retirement (decrement (a)), creeping senility (decrement (b)–already all too evident), and DEATH (decrement (c)). Assume that the corresponding "forces of mortality" are
  (a) $\mu^{(1)}(t) = 1/(10-t)$, $0 \le t \le 10$.
  (b) $\mu^{(2)}(t) = .03$.
  (c) $\mu^{(3)}(t) = 3t^2$.

  Write an expression for the probability that I cease work due to creeping senility within the next three years.

(3) A population of 1000 lives age 60 is subject to 2 decrements, disability (1), and death (2). You are given:
  (a)

x	$q_x^{\prime(1)}$	$q_x^{\prime(2)}$
60	0.02	0.04
61	0.04	0.06

(b) Decrements are uniformly distributed in each associated decrement.

Calculate the expected number of people who will become disabled before age 62.

(4) For a triple-decrement model, you are given the following information:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
37	0.02	0.02	0.05
38	0.05	0.05	0.10
39	0.05	0.05	0.16
40	0.07	0.06	0.02

Find

(a) ${}_2p_{37}^{(\tau)}$

(b) ${}_{2|}q_{37}^{(2)}$

(5) For a semi-continuous whole life insurance of 25,000 on (x), you are given:

(a) Expenses are paid at the beginning of the year and are as follows:

Yr.	% of Premium	Per 1000 of benefit	Per Policy
1	23%	3	14
Renewal	7%	0.47	2

(b) There is a settlement expense of 15 paid at the time of death.

(c) $i = 0.05$.

(d) Premiums are determined using the equivalence principle.

Give an equation that could be solved to find the level annual expense-loaded premium G.

(6) For a fully continuous whole life insurance on (30), you are given:

(a) The benefit is 3000 for death by accidental means (decrement 1).

(b) The benefit is 2000 for death by other means (decrement 2).

(c) ${}_tp_{30}^{\prime(1)} = e^{-t^2}$

(d) ${}_tp_{30}^{\prime(2)} = e^{-.03t}$

(e) $\delta = 0.05$

Give an expression that could be solved to find the benefit premium for this insurance.

(7) For a special fully discrete whole life of 10,000 on (x), you are given:
  (a) ${}_{10}AS = 1670$
  (b) ${}_{11}AS = 1650$
  (c) ${}_{11}CV = 1000$
  (d) $c_{10} = 0.05$ is the fraction of gross premium paid at time 10 for expenses.
  (e) $e_{10} = 80$ is the amount of per policy expense paid at time 10.
  (f) Death and withdrawal are the only decrements.
  (g) $q_{x+10}^{(d)} = 0.04$
  (h) $q_{x+10}^{(w)} = 0.03$
  (i) $i = 0.06$

  Give an expression that could be solved to find the gross premium G for this insurance.

(8) For a special 3-year term insurance, you are given:
  (a) Insureds may be in one of three states at the beginning of each year: active, disabled, or dead. All insureds are initially active. The annual transition probabilities are as follows:

	Active	Disabled	Dead
Active	0.7	0.2	0.1
Disbaled	0.5	0.3	0.2
Dead	0	0	1

  (b) A benefit B is payable at the end of each year in which the insured moves from active to disabled.

  Give an expression in terms of B and ν for the APV of this insurance.

(9) Let notation be as in problem (8). Assume that the insured pays P every year he/she is active at the beginning of years 0,1, and 2. Give an expression for the APV of the total payments in terms of P and ν.