MA 301 Test 4, Spring 2006

(1) State the "official" definition of "$\lim_{x\to a} f(x) = L$." 8 pts

(2) Find a value of a for which the following function is continuous at $x = 2$. Justify your answer. 8 pts

$$f(x) = \begin{cases} 2^{ax} & x > 2 \\ \sqrt{x} & 0 < x \leq 2 \end{cases}$$

(3) Use a δ-ϵ argument to prove that 12 pts

$$\lim_{x\to 3} \frac{x-1}{x+1} = \frac{1}{2}.$$

(4) Use a δ-ϵ argument to prove that 12 pts

$$\lim_{x\to 1} \frac{1}{\sqrt{2x+7}} = \frac{1}{3}.$$

(5) Use a δ-ϵ argument to prove that 12 pts

$$\lim_{x\to .5} \frac{1}{x^2} = 4.$$

(6) Assume that $\lim_{x\to a} f(x) = 5$. Use a δ-ϵ argument to prove that 12 pts

$$\frac{f(x)+3}{f(x)-1} = 2.$$

(7) Use a δ-ϵ argument to prove Theorem 3 on p. 180 of the notes: 12 pts

THEOREM 3 (Sequence). *Let $f(x)$ be continuous at a and let x_n be a sequence such that $\lim_{n\to\infty} x_n = a$. Then*

$$\lim_{n\to\infty} f(x_n) = f(a).$$