MA 266 Exam 1 Study Guide

Exam 1 will cover material from lessons 1-10. This is sections 1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, and 3.1 in the textbook.

Exam 1 will be worth 100 points. There are some multiple choice problems and some free response problems. There is no partial credit for multiple choice questions, but partial credit may be awarded for free response questions. You must show all of your work for full credit on the free response questions.

You should be able to do every homework question. Calculators are not allowed on the exam, so the computations will be simple enough to do by hand.

Terminology you should know (not an exhaustive list):

- differential equation
- direction field, slope field (same thing as direction field), equilibrium solution
- initial condition, initial value problem
- solution to a differential equation, solution to an initial value problem
- linear differential equation, nonlinear differential equation, order of a differential equation
- integrating factor
- separable equation, homogeneous equation ($\frac{y}{x}$ version), Bernoulli equation, autonomous equation, exact equation
- explicit solution, implicit solution, interval where the solution is valid
- mathematical model, gravity, air resistance, concentration (of some chemical in another chemical), etc.
- existence of a solution, uniqueness of a solution
- equilibrium solution, critical point, asymptotically stable, unstable, semistable, $f(y)$ vs. y graph, phase line, carrying capacity, threshold, logistic growth
- Euler's method
- homogeneous equation (0 "constant" term version), characteristic equation

Techniques you should be able to do (not an exhaustive list):

- Know how to graph a direction field, look at a direction field and describe behavior of solutions, identify which differential equation from a list produces a given direction field
- Identify a differential equation as ordinary vs. partial, linear vs. nonlinear, and the order of the differential equation

- Given a function, identify whether or not it is a solution to a given differential equation
- Know when to use the integrating factor method, compute the integrating factor $\mu(t)$, solve a differential equation with the integrating factor method
- Identify an equation as separable, solve a separable equation implicitly and explicitly (if possible), state where the solution to such an equation is valid
- Identify an equation as homogeneous ($\frac{y}{x}$) and solve a homogeneous equation with the substitution $u(x) = \frac{y}{x}$
- Be able to solve a differential equation given a substitution (see supplementary problems B and C)
- Model tank problems, model falling body problems, model Newton's Law of Cooling problems
- Understand and be able to apply Theorems 2.4.1 and 2.4.2 about the existence and uniqueness of solutions to initial value problems. Understand the limitations of Theorem 2.4.2 when compared to Theorem 2.4.1
- Identify a Bernoulli equation and solve a Bernoulli equation making the appropriate substitution
- Given an autonomous equation, graph $f(y)$ vs. y, graph the phase line, find equilibrium solutions, classify equilibrium solutions as asymptotically stable, unstable, or semistable, draw a qualitatively accurate sketch of several integral curves based on the phase line and equilibrium solutions, be able to talk about the carrying capacity and threshold of a population which can be modeled by an autonomous equation
- Show that a differential equation is exact, find the function $\psi(x, y)$ associated with an exact equation, solve an exact equation
- Understand how Euler's method works, use Euler's method to do some simple estimations of solutions to initial value problems
- Find the characteristic equation of second order linear homogeneous differential equations with constant coefficients, find the general solution of such a differential equation, solve an initial value problem containing such a differential equation
- Describe the behavior of solutions to a differential equation as $t \to \infty$, finding maxima and minima of solutions, finding critical initial values (values of a so that if $y(t_0) < a$, then the solution behaves in a very different way from if $y(t_0) > a$)

As was said earlier, this is *not* an exhaustive list of material that could appear on the exam – it is only a list of the biggest ideas covered. You should be capable of doing every homework problem (even the ones which are too long and/or difficult to be placed on an exam – I can still ask you how to do parts of these problems, even if I don't ask you to do the full problem).