MA 266 Final Exam Study Guide

The Final Exam will cover material from lessons 1-31. This is sections 1.1-1.3, 2.1-2.7, 3.1-3.8, 4.1-4.3, 6.1-6.6, 7.1-7.6, 7.8, and 7.9 in the textbook.

The final exam will be worth 200 points. There are 20 multiple choice questions. You will earn full credit for a correct response and 50% credit for an incorrect response with correct work. The exam questions will cover the entire summer term fairly evenly.

You should be able to do every homework question. Calculators are not allowed on the exam, so the computations will be simple enough to do by hand.

Terminology you should know (not an exhaustive list):

- differential equation
- direction field, slope field (same thing as direction field), equilibrium solution
- initial condition, initial value problem
- solution to a differential equation, solution to an initial value problem
- linear differential equation, nonlinear differential equation, order of a differential equation
- integrating factor
- separable equation, homogeneous equation ($\frac{y}{x}$ version), Bernoulli equation, autonomous equation, exact equation
- explicit solution, implicit solution, interval where the solution is valid
- mathematical model, gravity, air resistance, concentration (of some chemical in another chemical), etc. (See Lesson 5)
- existence of a solution, uniqueness of a solution
- equilibrium solution, critical point, asymptotically stable, unstable, semistable, $f(y)$ vs. y graph, phase line, carrying capacity, threshold, logistic growth
- Euler's method
- characteristic equation/polynomial
- linear operator
- Principle of Superposition
- Wronskian, fundamental set of solutions
- complex number, Euler's formula
- homogeneous, nonhomogeneous, complementary solution, particular solution
- Method of Reduction of Order
- Method of Undetermined Coefficients, Method of Variation of Parameters

- mass-spring system, Hooke's law, spring constant, spring force, gravity, damping force, damping coefficient, natural frequency, amplitude, phase, period, quasi-frequency, quasi-period, critically damped, overdamped, underdamped, free vibrations, forced vibrations, damped vibrations, undamped vibrations, forcing function, transient solution, steady state solution, beats, resonance, g, kg, cm, m, N (Newtons: $\mathrm{kg} \cdot \mathrm{m}/s^2$), lb, in, ft
- piecewise continuous function, Laplace transform
- unit step/Heaviside function $u_c(t)$, unit impulse/Dirac delta function $\delta(t)$, convolution integral, convolution of f and g (denoted $f * g$)
- system of first order linear differential equations, homogeneous system, nonhomogeneous system, matrix, real matrix, matrix addition, matrix multiplication
- vector, linearly independent set of vectors, linearly dependent set of vectors, eigenvalues, eigenvectors, characteristic equation of a matrix
- Phase plane, phase portrait, trajectories, node, saddle point, spiral point, center, improper node, asymptotically stable, stable, unstable
- generalized eigenvector $\boldsymbol{\eta}$
- Method of undetermined coefficients (for systems of equations)

Techniques you should be able to do (not an exhaustive list):

- Know how to graph a direction field, look at a direction field and describe behavior of solutions, identify which differential equation from a list produces a given direction field
- Identify a differential equation as linear vs. nonlinear and the order of the differential equation
- Given a function, identify whether or not it is a solution to a given differential equation
- Know when to use the integrating factor method, compute the integrating factor $\mu(t)$, solve a differential equation with the integrating factor method
- Identify an equation as separable, solve a separable equation implicitly and explicitly (if possible), state where the solution to such an equation is valid
- Identify an equation as homogeneous $(\frac{y}{x})$ and solve a homogeneous equation with the substitution $u(x) = \frac{y}{x}$
- Be able to solve a differential equation given a substitution (see supplementary problems B and C)
- Model tank problems, model falling body problems, model Newton's Law of Cooling problems
- Understand and be able to apply Theorems 2.4.1 and 2.4.2 about the existence and uniqueness of solutions to first order initial value problems. Understand the limitations of Theorem 2.4.2 when compared to Theorem 2.4.1

- Identify a Bernoulli equation and solve a Bernoulli equation making the appropriate substitution
- Identify an autonomous equation, given an autonomous equation, graph $f(y)$ vs. y, graph the phase line, find equilibrium solutions/critical points, classify equilibrium solutions as asymptotically stable, unstable, or semistable, draw a qualitatively accurate sketch of several integral curves based on the phase line and equilibrium solutions, be able to talk about the carrying capacity and threshold of a population which can be modeled by an autonomous equation
- Show that a differential equation is exact, find the function $\psi(x, y)$ associated with an exact equation, solve an exact equation
- Understand how Euler's method works, use Euler's method to do some simple estimations of solutions to initial value problems (if needed, the formula will be given)
- Describe the behavior of solutions to a differential equation as $t \to \infty$, finding maxima and minima of solutions, finding critical initial values (values of a so that if $y(t_0) < a$, then the solution behaves in a very different way from if $y(t_0) > a$)
- Find the characteristic equation of second order linear homogeneous differential equations with constant coefficients, find the general solution of second order linear homogeneous differential equations with constant coefficients when the characteristic polynomial has
    - Two distinct real roots
    - Complex conjugate roots
    - A repeated real root
- Compute the Wronskian of two functions, determine whether a set of functions is a fundamental set of solutions to a differential equation
- Write $c^{\lambda+\mu i}$ in $a + bi$ form for any positive real number c and real numbers λ, μ
- Use the method of reduction of order to find a second (algebraically distinct) solution to a second order linear homogeneous differential equation given one solution.
- Know how to solve nonhomogeneous second order linear differential equations using
    - Method of Undetermined Coefficients
    - Method of Variation of Parameters
- Set up IVPs representing the displacement of a mass attached to a spring, solve such IVPs, find the damping coefficient necessary for the system to be critically damped, overdamped, underdamped, and what such solutions look like, convert expressions of the form $A\cos(\alpha t) + B\sin(\alpha t)$ to the form $R\cos(\omega_0 t - \delta)$, find the natural frequency, amplitude, phase, and period of a mass-spring system, find the quasi-frequency and quasi-period of a mass-spring system, find the frequency of an external force for which the system will experience resonance or beats
- Determine intervals on which an nth order linear differential equation has solutions
- Solve an nth order linear homogeneous differential equation with constant coefficients
- Find the rational roots of any polynomial which has integer coefficients

- Be able to find a suitable form for $Y(t)$ using the Method of Undetermined Coefficients for a given nth order linear differential equation, solve an nth order linear nonhomogeneous differential equation with the Method of Undetermined Coefficients
- Determine whether a function is continuous, piecewise continuous, or neither
- Know the definition of the Laplace transform
- Compute the Laplace transform and inverse Laplace transform of functions we dealt with in chapter 6
- Solve IVPs with the Laplace transform, Solve IVPs with discontinuous forcing functions (using Heaviside step functions and/or Dirac delta functions)
- Compute the Laplace transform of convolution integrals, compute the inverse Laplace transform in terms of convolution integrals, solve IVPs in terms of convolution integrals
- Convert systems of first order linear differential equations into second order linear differential equations and vice versa
- Add, subtract, multiply matrices
- Determine whether or not a set of vectors is linearly independent
- Compute eigenvalues and associated eigenvectors of a matrix
- Solve 2×2 systems of differential equations when the coefficient matrix has distinct real eigenvalues, complex conjugate eigenvalues, repeated eigenvalues (including computing the generalized eigenvector)
- Classify the origin of the phase plane for a 2×2 system of linear differential equations in terms of stability and whether the origin is a node, saddle point, spiral point, center, improper node
- Describe the behavior of trajectories in a phase portrait
- Determine critical values α for which the qualitative nature of the phase portrait changes
- Solve nonhomogeneous 2×2 systems of differential equations using the method of undetermined coefficients.

As was said earlier, this is *not* an exhaustive list of material that could appear on the exam – it is only a list of the biggest ideas covered. You should be capable of doing every homework problem (even the ones which are too long and/or difficult to be placed on an exam – we can still ask you how to do parts of these problems, even if we don't ask you to do the full problem).