

High: 20
Low: 4
Average: 17

Math 266 Summer 2016 Quiz 11

1) Find all eigenvalues of the following matrix. For each eigenvalue,

find a corresponding eigenvector: $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$.

$$\begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda) + 2 = 6 - 5\lambda + \lambda^2 = (\lambda-2)(\lambda-3)$$

$$\lambda = 2, 3$$

For $\lambda = 2$

$$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix} \Rightarrow -x = -y \Rightarrow \text{Eigenvector } \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

For $\lambda = 3$

$$\begin{pmatrix} -2 & -1 \\ 2 & 1 \end{pmatrix} \Rightarrow -2x = -y \Rightarrow \text{Eigenvector } \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

2) Find the general solution to the following differential equation:

$$\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix} \mathbf{x}$$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & -4-\lambda \end{vmatrix} = (1-\lambda)(-4-\lambda) - 6 = \lambda^2 + 3\lambda - 10 = (\lambda+5)(\lambda-2)$$

$$\lambda_1 = 2, \lambda_2 = -5$$

$$\text{Eigenvector for } \lambda_1: \begin{pmatrix} -1 & 2 \\ 3 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\text{Eigenvector for } \lambda_2: \begin{pmatrix} 6 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\vec{x}(t) = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{-5t}$$