Math 266 Summer 2016 Quiz 2

1) A tank of 40 liters of water contains 2kg of salt at time 0, well-stirred. Water with a concentration of 1g of salt per liter flows into the tank at 2 liters/minute. The well-stirred mixture flows out at the same rate.
Write (but do not solve) a differential equation to find the amount of salt q in the tank t minutes. Include initial conditions.

$\frac{dq}{dt}$ = rate in − rate out

$= \frac{1g}{L} \cdot \frac{2L}{min} - \frac{q(t)g}{40L} \cdot \frac{2L}{min} = 2 - \frac{q(t)}{20}$

2kg = 2000 g

$q(0) = 2000$ grams

2) Solve the following initial value problem.

$\frac{dy}{dt} = 2y + 4 - t, \; y(0) = 0$

$\mu(t) = e^{-2t}$

$e^{-2t}\frac{dy}{dt} - 2e^{-2t}y + 4e^{-2t} - te^{-2t}$

$\frac{d}{dt}(e^{-2t}y) = 4e^{-2t} - te^{-2t}$

⇓ integrate

$e^{-2t}y = -2e^{-2t} + \frac{1}{2}te^{-2t} + \frac{1}{4}e^{-2t} + C$

$y = -\frac{7}{4} + \frac{1}{2}t + Ce^{2t}$

$y(0) = 0$

$\Rightarrow 0 = \frac{-7}{4} + C \Rightarrow C = \frac{7}{4}$

$y = -\frac{7}{4} + \frac{1}{2}t + \frac{7}{4}e^{2t}$

high: 20
Low: 11
Average: 15.5