Math 266 Summer 2016 Quiz 6

1) Rewrite $u = 2\sin(3t) - \cos(3t)$ in $u = R\cos(\omega_0 t - \delta)$ form.

$A\cos(\omega_0 t) + B\sin(\omega_0 t)$, $A = -1$, $B = 2$, $\omega_0 = 3$

$R = \sqrt{A^2 + B^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$

$\delta = \tan^{-1}\left(\frac{2}{-1}\right)$ in proper quadrant $= \tan^{-1}(-2)$

cos positive and sin negative happens in Q IV, so

don't need to add π $\boxed{\sqrt{5}\cos(3t + \tan^{-1}(2))}$

2) Suppose that a mass is attached to a spring and the position of the spring is given by the differential equation $3u'' + 75u = 0$. If an external periodic force $f(t) = 4\cos(\omega t)$ is then applied to the mass, for what value of ω would the mass-spring system experience resonance?

Resonance occurs when the external force has the same frequency as the natural frequency of the mass-spring system.

$3r^2 + 75 = 0$

$r^2 + 25 = 0$

$r^2 = -25$

$r = \pm\sqrt{-25} = \pm 5i$

$y_c(t) = c_1 \cos(5t) + c_2 \sin(5t)$

$\omega_0 = 5$

so $\boxed{\omega = 5}$ is required

for resonance

high: 18
low: 8
mean: 16.14