Math 266 Summer 2016 Quiz 9

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
2.	$\sin at$	$\frac{a}{s^2+a^2}$
3.	$\sinh at$	$\frac{a}{s^2-a^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$
3.	$\delta(t-c)$	e^{-cs}

1) Solve the initial value problem:

High: 20
Low: 7
Average: 15

$$y^{(4)} - y = \delta(t-1)$$

$$y(0) = y'(0) = y''(0) = y'''(0) = 0$$

Full solution on back

$$y(t) = \frac{1}{2} u_1(t)\left[-\sin(t-1) + \sinh(t-1)\right]$$

2) Find the Laplace transform of

$$f(t) = \begin{cases} 0, & t < 3 \\ (t-3)^5, & t \geq 3 \end{cases}$$

$$f(t) = (t-3)^5 u_3(t)$$

$$\mathcal{L}\left[(t-3)^5 u_3(t)\right] = e^{-3s}\mathcal{L}\left[t^5\right] = \frac{5!\, e^{-3s}}{s^6}$$

1. $\mathcal{L}\{y^{(4)}\} - \mathcal{L}\{y\} = \mathcal{L}\{\delta(t-1)\}$

$$s^4 Y(s) - s^3 y(0) - s^2 y'(0) - s y''(0) - y'''(0) - Y(s) = e^{-s}$$

$$(s^4 - 1) Y(s) = e^{-s}$$

$$Y(s) = \frac{e^{-s}}{s^4 - 1} = \frac{e^{-s}}{(s^2+1)(s^2-1)}$$

$$\frac{1}{(s^2+1)(s^2-1)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2-1} \Rightarrow 1 = (As+B)(s^2-1) + (Cs+D)(s^2+1)$$

$$\Rightarrow \begin{aligned} A + C &= 0 \\ B + D &= 0 \\ -A + C &= 0 \\ -B + D &= 1 \end{aligned} \quad \Rightarrow \quad C = 0,\ A = 0,\ D = \frac{1}{2},\ B = \frac{-1}{2}$$

$$Y(s) = e^{-s}\left(-\frac{1}{2}\left(\frac{1}{s^2+1}\right) + \frac{1}{2}\left(\frac{1}{s^2-1}\right)\right)$$

$$\Rightarrow \quad y(t) = \frac{1}{2} u_1(t) f(t-1)$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{-1}{s^2+1} + \frac{1}{s^2-1}\right\} = -\sin(t) + \sinh(t)$$

so

$$y(t) = \frac{1}{2} u_1(t)\left[-\sin(t-1) + \sinh(t-1)\right]$$