MA 595. FILTERING COMPLEX FLUID SYSTEMS (SPRING 2025)

INSTRUCTOR: DI QI (EMAIL: QIDI@PURDUE.EDU)

Time: Mon. Wed. Fri. 09:30 - 10:20 AM, Helen B. Schleman Hall 112
First Class: January 13, 2025
Last Class: May 3, 2025

For other important dates, see Purdue 2024-2025 Academic Calendar.
(see also `Course Webpage` for supplementary course materials)

Course Description:

This advanced topic course will discuss filtering noisy turbulent signals for complex dynamical systems through an applied mathematics perspective involving the blending of rigorous mathematical theories, qualitative and quantitative modeling, and novel numerical procedures. The course will begin with an elementary introduction to these topics including classical analysis for PDEs and SDEs and their finite computational approximations, data assimilation methods including Kalman filtering, ensemble Kalman filters, and instructive stochastic qualitative models from turbulence theory and concrete models such as from climate atmosphere ocean science. Recent development in new mathematical theories and algorithms for fully nonlinear dynamical systems will also be discussed.

Prerequisite: Basics of ODEs, PDEs, and stochastic processes, experiences in computational methods and writing computer programs are preferred but not necessary.

Audience: The course should be suitable to any graduate students and postdocs in applied mathematics, physics, engineering, and climate, atmosphere, ocean sciences as well as related fields interested in filtering and data assimilation for complex turbulent dynamical systems.

Major Learning Outcomes:

- ➢ Filtering theory:
  (1) Martingales, stochastic integrals, and stochastic differential equations
  (2) Filtering model: Zakai and Kushner-Stratonovich equations
  (3) Uniqueness of the solution for the filtering equations
  (4) Stability of linear and nonlinear filtering
- ➢ Numerical algorithms:
  (1) Kalman filter and extended Kalman filters
  (2) Monte-Carlo method and particle filter
  (3) Filters in continuous and discrete time
- ➢ Applications:
  (1) Filtering turbulent signals
  (2) Filtering stochastic PDEs with nonlinearity and instability
  (3) Using software tools such as visualization and specialized computing packages.

Learning Resources & Course Arrangements:

Reference books and reading materials:

- ➢ Majda, A.J., & Harlim, J. (2012). *Filtering Complex Turbulent Systems.* New York, NY: Cambridge University Press.
- ➢ Xiong, J., (2008). *An introduction to stochastic filtering theory (Vol. 18).* OUP Oxford.
- ➢ Bain, A. and Crisan, D., (2009). *Fundamentals of stochastic filtering (Vol. 3).* New York: Springer.
- ➢ Liptser, R.S. and Shiryaev, A.N., (1977). *Statistics of random processes: I. General theory (Vol. 394), II: Applications (Vol. 6).* New York: Springer-verlag.

Course Resources: The main learning management system will be *Brightspace*. Lecture notes will be posted on *Brightspace*. The following platforms (accessible through *Brightspace*) will also be used for assignments and other activities:

- ➢ *Piazza* for class discussions.
- ➢ *Zoom* for potential online discussions and virtual meetings.

Course Schedule:

Week	Dates	Topics	Comments
1	Jan. 13, 15, 17	Introduction, basic definitions, and filtering equations	
2	Jan. 22, 24	Filtering stochastic scalar and vector systems	No class on MLK Day
3	Jan. 27, 29, 31	Reduced filters and multiscale slow-fast systems	
4	Feb. 3, 5, 7	Martingales, stochastic integrals and SDEs	
5	Feb. 10, 12, 14	Zakai and Kushner-Stratonovich equations	
6	Feb. 17, 19, 21	Uniqueness of filtering solutions	
7	Feb. 24, 26, 28	Robust representation formula	
8	Mar. 3, 5, 7	Linear filtering: Kalman-Bucy filter	
9	Mar. 10, 12, 14	Nonlinear filtering: extended Kalman filters	
	Mar. 17-22		No class this week (Spring Break)
10	Mar. 24, 26, 28	Monte-Carlo method and particle systems	
11	Mar. 31, Apr. 2, 4	Stochastic models for turbulence	
12	Apr. 7, 9, 11	Filtering turbulent signals: regular and sparse observations	
13	Apr. 14, 16, 18	Gaussian filtering strategies for nonlinear systems	
14	Apr. 21, 23, 25	Nonlinear filtering strategies	
15	Apr. 28, 30, May 2	Student presentations	

(Schedule is subject to change. Any changes will be posted on *Brightspace*.)

Lecture Notes: Lecture notes will be posted in *Brightspace*. Typically, there will be three lectures per week, each corresponding to a section in the textbook and about 50 mins long.

Office Hours: WF, 12:00-1:00 pm (Eastern Time) or through appointment. The time is subject to change. The information will be posted on *Brightspace*.

Assignments and grading: The students will be assigned one research project, which will involve either theoretical analysis or programming tasks, and preparing a typeset report (preferably in LaTex). You may discuss strategies to solve the problems with peers, but everything in the derivation, proofs and every line of code (including figures) must be 100% yours. Allowing others to copy solutions or codes from you is considered cheating. The students working on the research project are expected to present their results in the last week of the class.

There will be no homework assignment and exams. The grade will be based on the project and class attendance.

This semester, we will apply the following rule: students who get at least 97% of the total points are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77%a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-. Please note, these are not the actual cutoffs, but rather upper bounds on those. The actual cutoffs of these grades will be determined after the final exam and can be lower but not higher than the ones above. Thus, the actual cutoff for A can be, say 85%, but not 95%.

Course and University Policies:

Attendance and course engagement: This is an in-person course. The students are expected to attend the lectures and read the corresponding lecture notes. Students are responsible for completing and submitting all assignments on time. Students need to give presentation on their project in the last week of the semester. Students are encouraged to participate actively during the lectures and after-class discussions. Students must also check periodically for possible changes in the course schedule (on *Brightspace*) including due dates for assignments and exams.

Late homework: Late homework will not be accepted. In the event that an assignment is missed for reasons that are serious, unavoidable, and beyond the student's control, the situation will be handled on an individual basis. Documentation may be required in such cases.

Makeup policy: Student needs to inform the instructor of any conflict that can be anticipated and will affect the submission of an assignment or the ability to give the presentation. Only the instructor can excuse a student from a course requirement or responsibility. When conflicts can be anticipated, such as for many University-sponsored activities and religious observations, the student should inform the instructor of the situation as far in advance as possible. For unanticipated or emergency conflict, when advance notification to an instructor is not possible, the student should contact the instructor as soon as possible by email, through *Brightspace*, or by phone. When the student is unable to make direct contact with the instructor and is unable to leave word with the instructor's department because of circumstances beyond the student's control, and in cases of bereavement, quarantine, or isolation, the student or the student's representative should contact the Office of the Dean of Students via email or phone at 765-494-1747. Our course *Brightspace* includes a link on Attendance and Grief Absence policies under the University Policies menu.

Academic integrity: Academic integrity is expected for all students at all times in this course. You are free (even encouraged) to work with other students to solve the homework problems. However, you are required to complete and write up solutions for the homework using your own words and on your own. If you worked with any humans, book, the internet, you should be explicit about it and list all sources and the extent of help you got from each resource (no points will be taken for such disclosures). But if you present as your own work what was not, then you will get zero points on the assignment and an academic misconduct filing after the first instance.

Nondiscrimination statement: Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. More details are available on our course *Brightspace* table of contents, under University Policies.

Students with disabilities: Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are welcome to let me know so that we can discuss options. You are also encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247. In this mathematics course accommodations are managed between the instructor, the student and DRC Testing Center. If you have been certified by the Disability Resource Center (DRC) as eligible for accommodations, you should contact your instructor to discuss your accommodations as soon as possible. Here are instructions for sending your Course Accessibility Letter to your instructor: https://www.purdue.edu/drc/students/course-accessibility-letter.php.

Emergency preparation: In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors or TAs via email or phone. You are expected to read your *@purdue.edu* email on a frequent basis.

COVID-19 statement: If you become quarantined or isolated at any point in time during the semester, in addition to support from the Protect Purdue Health Center, you will also have access to an Academic Case Manager who can provide you academic support during this time. Your Academic Case Manager can be reached at acmq@purdue.edu and will provide you with general guidelines/resources around communicating with your instructors, be available for academic support, and offer suggestions for how to be successful when learning remotely. Importantly, if you find yourself too sick to progress in the course, notify your academic case manager and notify me via email or *Brightspace*. We will make arrangements based on your particular situation. The Office of the Dean of Students (odos@purdue.edu) is also available to support you should this situation occur. Other important policie scan be found in *Brightspace*.

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