

OVERVIEW OF RESULTS AND ACTIVITIES OF RALPH M. KAUFMANN

1. RESULTS IN MATHEMATICS

1.1. **Selected results.** Highlight results are:

- (1) The definition and recursion relation for "Higher–Weil–Petersson" volume for moduli spaces together with Y. Manin and D. Zagier [57]. .
- (2) The explicit definition of the Quantum Cohomology of a product and its Künnth formula, [56,58], the first paper which is coauthored by M. Kontsevich and Y. Manin
- (3) The definition of stringy K–theory and Chern classes, coauthored with T. Jarvis and T. Kimura, [41], which is a fundamental result in the theory of stacks.
- (4) The proof a the cyclic version of Deligne’s conjecture on Hochschild cohomology as conjectured by Tamarkin-Tsygan [37]. This is part of a whole slew of general results related to algebraic string topology operations.
- (5) The categorical formulation and generalization of operad–like theories [19] together with my student B. Ward.

Below is a longer list organized area.

1.2. **Algebraic/Complex Geometry.** Going back to my thesis [55], I have studied the Knudsen-Deligne-Mumford compactifications of the moduli spaces of pointed Riemann surfaces $\bar{M}_{g,n}$. Moduli spaces and mirror symmetry have been an international area of research for the last three decades.

- (1) I gave an intersection formula for the cohomology rings (or Chow rings) $H^*(\bar{M}_{0,n})$, established by Manin and Kontsevich [58]. This is a basic result of the theory and establishes an explicit cup product and a Künneth formula for quantum cohomology, [56].
- (2) We, in collaboration with Manin and Zagier [57], defined generalizations of the classical Weil-Petersson volumes, the so-called higher Weil-Petersson (HWP) volumes and gave generating functions for them. This is a start of a whole research activity. The higher genus version is equivalent to the Witten’s gravitational decedents as shown by Zograf and Manin, and these considerations were generalized further by M. Mirzarkhani. The structure of the recursion formulas can be seen as a precursor to topological recursion.
- (3) We, with T. Kimura and T. Jarvis, gave the fundamental definition of stringy K–theory and a string Chern character [41]. This is based on prior work on stacks [45], which was also used by Costello in his definition of higher genus Gromov–Witten Invariants.
- (4) Further results concern global stringy theories, like deRham theory and twisted K–theory [31], Hilbert schemes [49] and singularities with symmetries, [44] which was the foundation for the Witten-Fan-Jarvis-Ruan theory for singularities.
- (5) We in collaboration with B. Ward and J. Zuniga has given the algebraic, complex geometric background of Master Equations [57]. These have their origin in statistical processes, but also in string field theory following Zwiebach. Recently, J. Zuniga and I [1] gave real blowup versions of moduli spaces based on graphs as they appear in the theory of Strebel differentials.

1.3. **Algebra.** My algebraic results have three sources, the first is from orbifolds and concerns Frobenius algebras and Drinfel'd doubles, the second are Hopf algebras as they appear in renormalization in the Connes–Kreimer formalism and in incidence algebras as studied by Joni and Rota, and the third are infinite dimensional algebras and their representation as they appear in conformal field theory.

- (1) The resulting basic algebra and the related mirror symmetry operation was described in [52, 53]. In [48], I proved a uniqueness and existence result for twisted symmetric powers. I introduced the Drinfel'd double and its cousin, Drinfel'd Yetter modules into the theory [47] and together with my student, D. Pham considered twisted version [35], simplifying the axioms and explaining the phenomenon of discrete torsion on the cohomological, bundle and K-theoretic levels.
- (2) In [13, 14], three famous Hopf algebras from several different sources, the Hopf algebra of Goncharov for multi-zeta values in number theory, the Hopf algebra of Baues for the chains on a double loop space in algebraic topology, and the Connes–Kreimer Hopf algebras for renormalization are put into a common framework. There are three levels, simplicial, cooperadic treated in the first paper and categorical in the second paper. In a follow-up work, together with my student Y. Mo, the type of Bi- and Hopf algebras that appear in this context are characterized [9] and the conditions for them to be Hopf algebras were given, as well as a quantum localization construction.
- (3) Together with L. Borisov, [29], we gave a vertex operator algebra version of $N = (0, 2)$ super-symmetry, which is at the heart of the Calabi–Yau/Landau–Ginzburg correspondence. Previously, in [59] RK considered infinite dimensional representations of the Virasoro algebra and made headway in proving Nahm's conjecture on signature characters.

1.4. **Algebraic topology.** My algebraic topology results are centered around action on the Hochschild complex that are extensions of Deligne's original conjecture on the Hochschild complex which states that there should be a chain model of the little discs operad, which acts on Hochschild cochains of an associative algebra, in such a way that this action on the homology/cohomology level is the Gerstenhaber structure. One application of these results is given by Kontsevich's famous deformation quantization theorem which as Tamarkin pointed out follows from Deligne's conjecture and the formality of the little discs operad.

- (1) Besides the aforementioned cyclic version [37] for a Frobenius algebra, I gave a correct descriptive proof of the conjecture in [42] and linked it to Hopf algebras. The A_∞ version is proven explicitly together with my student R. Schwell in [34] using polytopes.
- (2) I gave String topology operations were given by RK in [39, 40], where the first paper deals with the subtleties of the topological operations and the second defines the operations on Hochschild complexes. This includes the product and bracket operations of Chas and Sullivan and the operation of a coproduct made famous by the constructions of Goresky and Hingston [8]. The most general relations stem from a compactification of Moduli Space of Riemann surfaces. The theory was generalized to the open/closed case in [32].
- (3) In straight-up algebraic topology I have worked with A. Medina-Mardones [7, 10] on cubical chains and Steenrod operations, where we prove new explicit results in this venerable context.

1.5. Differential geometry, Foliations and Teichüller theory. On the differential topological side, I worked on Frobenius manifolds which by results of Dubrovin and Manin are the differentiable structure organizing quantum cohomology and on foliations as they appear in Thurston's description of Teichmüller space.

- (1) I defined the Künneth tensor product in the setting of Frobenius manifolds previously only locally defined via germs, by using an affine connection this can be used to define a global version of a tensor product [54].
- (2) Together with Muriel Livernet and Bob Penner, we pioneered a method of defining operation on foliations of surfaces in [43, 51]. Followups pertaining to physics are in [38] and the theory is in the Handbook of Teichmüller theory [26]. There are chain level and homological versions of this, which are what is at the heart of the algebraic operations above.
- (3) The restriction to a particular subspace yields the proof of a claim by Voronov which announces the equivalence of the framed little discs operad and his cacti operad [46] which plays a fundamental role in string topology. The little cubes structure given by spineless cacti Cacti is related to permutahedral complexes, which appear in the work of Milgram, Berger and Blagoevich-Ziegler in a concrete geometric way as shown together with my student Y. Zhang [20].
- (4) In a similar, but different direction, I could show that a natural geometric stabilization, not unlike the one used by Tillmann/Madsen/Weiss to prove Mumford's conjecture, yields a model for all little cubes operads [36] and hence an infinite loop machine. Algebraically this acts on the Hochschild complex of a semi-simple Frobenius algebra providing an E_∞ structure [38].

1.6. Category theory. The main contribution in this field is the categorical formulation generalizing operads, PROPs, modular operads. This theory was initiated by Mac Lane to organize structures appearing in algebraic topology and was popularized by Stasheff and May through operadic recognition principle for loop spaces and furthered by Ginzburg, Getzler and Kapranov.

- (1) The basic theory is developed with my student B. Ward in [19], with aspects elucidated in [17]. This is a full treatise with basic definitions and results.
- (2) Follow up work connecting it to logic is in [18], to representation theory in [11], to Koszul duality in [6, 69] and to Moduli Spaces and Conant-Vogtman Outer Space in [67].
- (3) RK and his student M. Monaco developed a new way of regarding categories in terms of categorical bimodule monoids. This has led to a generalization of Baez-Dolan type plus constructions and other fundamental results in the area of operads and categories, [3, 68].

1.7. Mathematical Physics: topological and geometric properties of materials. Besides the string and conformal field theory inspired mathematics, I made contributions to condensed matter physics with methods from singularity theory, C^* -theory, K-theory and representation theory. Here I worked in two groups, one with B. Kaufmann and S. Khlebnikov from physics and one with B. Kaufmann and D. Li, their common post-doc.

- (1) The most important contribution is in [28], an Annals of Physics paper, where we use singularity theory to classify topological defects.

- (2) Prior to this we used classical and C^* geometry to analyze triply periodic materials and their associated nano-wire networks, [27, 30] and gerbes in [23]. This is reviewed for material scientists in [16].
- (3) The differential geometric and analytic setup of the theory is developed in [12, 22].
- (4) With the second team, we wrote the well-cited article [21] which surveys, recasts and adds results to the theory of topological insulators and their topological invariants. In the recent paper [4], we give a complete and precise mathematical description of topological insulators with time-reversal or particle-hole symmetries using Real K-theory.

1.8. **Applied Mathematics.** I have also recently co-authored papers in applied mathematics, collaborating with physics, chemistry, engineering and medicine.

- (1) Together with my student M. Mousa, B. Kaufmann, S. Cui and S. Kais from Chemistry there is a project on topological phase transition with the first paper being [70].
- (2) In a collaboration that started with D. Trang-Duy from Purdue Mechanical Engineering whom I trained in TDA, we co-wrote a paper as first authors in neuroscience [5] which has the follow up [2].

2. SERVICE

2.1. **At the department/university level.**

- (1) I have mentored five post-docs in several areas of mathematics: Algebraic geometry, symplectic geometry, algebraic topology, algebra and mathematical physics. All of them have gone on to academic jobs, most of them to tenure track positions.
- (2) I have graduated 11 students in the fields of algebraic topology, algebraic geometry, algebra, Teichmüller theory and topological quantum field theory. All of his students were offered post-doctoral positions and many remained in academia.
- (3) I taught the full gamut of courses and offered reading courses and mentoring to students at all levels. This includes large calculus courses, the topology sequence, and algebraic geometry sequence (at Uconn) and topics courses ranging from stacks, K-theory, index theorems and genera, Characteristic classes, Hopf-algebras to topological data analysis and quantum computing.
- (4) I have served on a large number of committees including the senate, dedicating a large amount of my time.

2.2. **For the mathematical community at large.**

- (1) In my opinion, my most important contribution is the co-founding of the journal “Higher Structures”. This growing journal is indexed in mathscinet and Zentralblatt.
- (2) I has organized a lengthy list of conferences including a trimester on “Higher structures” and an interdisciplinary month-long program on “Combinatorics, Resurgence and Algebraic Geometry in Quantum Field Theory” at the MPI.
- (3) I have refereed for the top international journals and international funding agencies.

2.3. **Talks and research stays.**

- (1) I have given 161 colloquium and seminar talks and 89 conference talks.
- (2) I have has received multiple invitations to stays at the worlds leading research institutions, like the IAS in Princeton, the IHES in Paris, the MPI in Bonn, the MSRI in

Berkeley, the Newton Institute in Cambridge, the CRM in Barcelona and the Field Institute in Toronto.

- (3) I have just received a semester invitation to the Hamburg Institute for Advanced Studies which is an interdisciplinary research center.

3. ACTIVITIES IN LIBERAL ARTS

I do not only work with physicists, where I hold a masters degree and a courtesy appointment, and STEM fields, but also with liberal arts, where I hold a masters degree and a courtesy appointment in philosophy.

- (1) I published five papers on Hegel, four of them joint with C. Yeomans, [71, 72, 75, 77, 79] two of them published in Cambridge University Press Guides.
- (2) I published a paper on Humboldt and his epistemological background [73] and presented these results at a conference on Romanticism.
- (3) I have also published about classical German poetry [78] which is published with the flagship German philosophy publisher Meiner, and about prize winning contemporary German language poets [76, 80, 82].
- (4) I have separately published reflections on mathematical language in the IAS newsletter and on thinking [74]
- (5) In this capacity, I have been invited to short stays at the Stiftung Homborich (an arts foundation in Germany) and the prestigious Villa Massimo, the German equivalent of the American Academy in Rome.

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Mit Beiträgen von Ralph Kaufmann und Oswald Egger. (With contribution by)
Edition: Das böhmische Dorf 2006.