Homework 1

MA 35100 (Spring 2025, §§130-131)

January 13th, 2025

Instructions

- Due: Saturday, January 25th at 11 PM Eastern Time.
- Total Score: 25 points.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

The following is relevant for Problems 1, 2 and 3:

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 \\ -1 & 2 \\ 1 & 2 \\ 3 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Problem 1. [3 points] For which pairs of matrices X and Y from $\{A, B, C, D\}$ is the sum $X + Y$ well-defined? Compute those sums.

Problem 2. [5 points] Which of the matrices A, B, C, and D are in echelon form? For those which are not in echelon form, compute an echelon form matrix which is row-equivalent to it.

Problem 3. [5 points] Which of the matrices A, B, C, and D are in reduced row echelon form? For those which are not in reduced row echelon form, compute a reduced row echelon form matrix which is row-equivalent to it.

Problem 4. [3 points] Solve the following system of equations:

$$\begin{aligned} x + 2y + 3z &= 4 \\ 5x + 6y + 7z &= 8 \\ 9x + 10y + 11z &= 12 \end{aligned}$$

Problem 5. [3 points] Solve the following system of equations:

$$\begin{aligned} 9x + 10y + 11z &= 13 \\ 5x + 6y + 7z &= 8 \\ x + 2y + 3z &= 4 \end{aligned}$$

Problem 6. [3 points] Solve the following system of equations:

$$\begin{aligned} x + y + z &= 1 \\ 2x + 2y + z &= 0 \\ 3x + 5y + 7z &= -11 \end{aligned}$$

Problem 7. [3 points] Which of the following sets are vector spaces?

1. The set of all integers $\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$.
2. The set of all polynomials $p(x)$ satisfying $p(0) = 0$.
3. The set of all points $(x, y) \in \mathbb{R}^2$ that lie on the line $x + y = 1$.

For those that are not vector spaces, give concrete examples of one of the vector space axioms failing. For those that are vector spaces, give a justification for why the set is closed under addition and scalar multiplication.