Homework 4

MA 35100 (Spring 2025, §§130-131)

February 7th, 2025

Instructions

- Due: Saturday, February 15th at 11 PM Eastern Time.
- Total Score: 25 points.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

The following is relevant for Problems 1, 2, 3, and 4:

$$A_1 = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 0 & 1 \\ 5 & 0 \\ 10 & 2 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 0 & 0 \end{bmatrix}.$$

$$\vec{b}_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \qquad \vec{b}_2 = \begin{bmatrix} 5 \\ 10 \end{bmatrix}, \qquad \vec{b}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

Problem 1. [6 points] For which pairs $(A, \vec{b})$ with $A \in \{A_1, A_2, A_3, A_4\}$ and $\vec{b} \in \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ is the product $A\vec{b}$ well-defined? Compute the ones that are well-defined.

Problem 2. [8 points] For each $A \in \{A_1, A_2, A_3, A_4\}$, describe the nullspace, $\mathrm{Null}(A)$ as a span.

Problem 3. [5 points] For each $A \in \{A_1, A_2, A_3, A_4\}$, determine the rank r, the number of rows m, and whether $\mathrm{Col}(A) = \mathbb{R}^m$. Do you see a pattern in your answers?

Problem 4. [6 points] For which pairs $(A, \vec{b})$ with $A \in \{A_1, A_2, A_3, A_4\}$ and $\vec{b} \in \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ does it makes sense (due to dimension constraints) to ask if $\vec{b} \in \mathrm{Col}(A)$? For those pairs for which it makes sense, determine the answer to the question $\vec{b} \in \mathrm{Col}(A)$.