Homework 6

MA 35100 (Spring 2025, §§130-131)

February 22nd, 2025

Instructions

- Due: <u>Wednesday, March 5th</u> at 11 PM Eastern Time. **Note:** non-standard day!

- Total Score: 25 points.

- The three lowest homework scores will be dropped from the final grade.

- One late submission is permitted (over the course of the semester) with no questions asked.

- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.

- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

The following is relevant for all the problems in this homework set:

$$X_1 = \begin{bmatrix} 1 & 2 & 0 & -2 \\ 2 & 3 & 2 & 3 \\ 2 & 10 & 4 & 10 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \end{bmatrix}, \qquad X_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$X_4 = \begin{bmatrix} 4 & 9 \\ 1 & 2 \end{bmatrix}, \qquad X_5 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}, \qquad X_6 = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}.$$

Problem 1. [15 points] For which $A, B \in \{X_1, X_2, X_3, X_4, X_5, X_6\}$ is the matrix product AB well-defined? If it is well-defined, compute it. Note that you need to include the cases where A and B are the same matrix X_j, and further, since matrix multiplication is noncommutative, you need to consider the case $A = X_i$ and $B = X_j$ as well as the case $A = X_j$ and $B = X_i$.

Problem 2. [10 points] For which $X \in \{X_1, X_2, X_3, X_4, X_5, X_6\}$ is X nonsingular? For those that are, compute the inverse matrix X^{-1}.