Homework 7

MA 35100 (Spring 2025, §§130-131)

March 4th, 2025 (updated: March 9th)

Instructions

- Due: ~~Wednesday, March 12th~~ Friday, March 14th at 11 PM Eastern Time. **Note:** non-standard day!
- Total Score: 25 points.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [2 points] Let T, S, and R be defined by the expressions

$$T(x, y) = x/y, \qquad S(x, y) = x^2 + y^2, \qquad R(x, y) = x + y.$$

Determine for $F \in \{T, S, R\}$ whether $F : \mathbb{R}^2 \to \mathbb{R}$ is a well-defined transformation.

Problem 2. [2 points] For $F \in \{T, S, R\}$ as in the previous problem, if F is well-defined, determine if F is a linear transformation.

Problem 3. [6 points] Let $V = W = P_n(\mathbb{R})$ for some $n \geq 1$. Further, suppose one wanted to define transformations $D : V \to W$ and $I : V \to W$ with the rules

$$D(p(t)) = p'(t), \qquad I(p(t)) = \int_0^t p(x)dx.$$

For $F \in \{D, I\}$, is F a well-defined transformation? If not, can you suggest a change to the codomain W, so that $F : V \to W$ would be well-defined? Are the resulting transformations linear?

Problem 4. [5 points] Define $S \subset \mathcal{C}(\mathbb{R})$ to be the set of constant functions. That is, $p \in S$ implies for some fixed $c \in \mathbb{R}$ and every $t \in \mathbb{R}$, $p(t) = c$. What is the image of S under the maps D and I from above?

Problem 5. [10 points] Let C be the unit circle in $\mathbb{R}^2$. That is,

$$C = \{(x, y) : x^2 + y^2 = 1\}.$$

Determine $T_j(C)$, the image of C under the ~~linear~~ transformation $T_j : \mathbb{R}^2 \to \mathbb{R}^2$, where T_j, $j = 1, 2, 3, 4$ are given below.

- $T_1(x, y) = (x, 0)$.
- T_2 is an anticlockwise rotation around the origin by an angle of $\pi/4$ radians.
- $T_3(x, y) = (x, 2y)$.
- T_4 is a reflection along the line $x + y = \sqrt{2}$.