MA 35100: Elementary Linear Algebra

Sample Midterm 1

February 25th, 2025

- This is the exam for §§130-131 of MA 351.
- The exam will be 60 minutes long.
- There are 15 pages.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use theorems proved in class provided you accurately state them before using them.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- When the time is over, put down your writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ____________________

PUID: ____________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ____________________

QUESTION	VALUE	SCORE
1	15	
2	5	
3	5	
4	5	
5	5	
6	5	
TOTAL	40	

1. (15 points)

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) The dimension of the space $M_{2\times 3}(\mathbb{R})$ is 6.

(b) Any homogeneous system of linear equations with 2 equations and 4 variables has infinitely many solutions.

(c) The subset $W = \{(x, y) : x, y \in \mathbb{R} \text{ and } xy = 0\}$ is a subspace of $\mathbb{R}^2$.

(d) If V is a vector space, W a subset, and the zero vector $0_V \in W$, then W must be a subspace.

(e) If n, r and $r^\#$ have their usual meanings for a linear system, then we always have $r \leqslant n, r^\#$.

(f) If A and B are row-equivalent, then they have the same nullspace.

(g) The matrix $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form.

(h) The set of matrices A in $M_{2\times 3}$ with the property that $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \vec{0}$ is a subspace of $M_{2\times 3}$.

(i) For any vector space V and any $\vec{v} \in V$, $1\vec{v} = \vec{v}$.

(j) The set of polynomials $\{t^2 + t + 1, 2t^2 + 3t + 1, t - 1\}$ is linearly independent.

2. (5 points)

For the following two statements determine whether they are true or false. In either case, provide appropriate justification for your answer (e.g., a counterexample or a brief proof).

(a) The space of skew-symmetric 3×3 matrices $\{A \in M_{3\times 3}(\mathbb{R}) : A^T = -A\}$ has dimension 6.

(b) Suppose that $a \neq 0$. Then, the matrix 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is nonsingular if and only if $ad - bc \neq 0$.

(Continued)

3. (5 points) For what values of k, ℓ does the following system have no solution, a unique solution, and infinitely many solutions respectively?

$$\begin{aligned} x + y + z &= 2 \\ x + 2z &= 3 + k \\ x - y + \ell z &= 2 \end{aligned}$$

Final Answer:

(Continued)

4. (5 points) Find all solutions to the following system of equations.

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 3x_1 - x_2 + 2x_3 - x_4 &= 7 \\ -x_1 - 2x_2 - x_3 - 2x_4 &= 2 \\ x_2 + x_4 &= -2 \end{aligned}$$

Final Answer:

(Continued)

5. (5 points)

Find a basis for Span S, where

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 7 \\ 11 \\ 13 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix} \right\}.$$

Final Answer:

(Continued)

6. (5 points) Compute the rank and nullity of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ -4 & 0 & 4 & -8 & 4 \end{bmatrix}$$

Final Answer:

(Continued)

Scratch Work

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