MATH 142

Final

December 16, 2002

NAME (please print legibly): _____

Your University ID Number: _____

Circle your Instructor's Name along with the Lecture Time:

Zokhrab Moustafaev (MWF 9:00 - 9:50) Carl Mueller (MW 3:25 - 4:40)

- **No calculators are allowed on this exam.**
- **You must do both parts of the final. The first part can make up for a bad midterm grade, but the midterms cannot make up for the first part of the final.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your final answers in the spaces provided.**

Part A		
QUESTION	VALUE	SCORE
1	16	
2	14	
3	20	
4	16	
5	24	
6	20	
7	20	
8	20	
TOTAL	150	

Part B		
QUESTION	VALUE	SCORE
9	16	
10	28	
11	18	
12	36	
13	8	
14	16	
15	8	
16	10	
17	10	
TOTAL	150	

Part A

1. (16 pts) Let

$$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x$$

(a) (5 points) Find the intervals on which $f(x)$ is increasing and decreasing.

(b) (5 points) Find the local extrema of $f(x)$.

(c) (6 points) Find the intervals on which f is concave up and concave down.

2. (14 pts)

Let

$$y = \frac{x^2 - 2x + 2}{2x^2 - 5x + 3}.$$

(a) (7 points) Find the vertical asymptotes.

(b) (7 points) Find the horizontal asymptotes.

3. (20 pts)

A box with a square base and open top must have a volume $32m^3$. Find the dimensions of the box that minimizes the amount of material used.

4. (16 pts)

Differentiate the following functions.

(a) (8 points)

$$\int_x^0 e^{-2t^2}\,dt$$

(b) (8 points)

$$\int_0^{x^3} \sin(t^2)\,dt$$

5. (24 pts)

Evaluate the following integrals.

(a) (8 points)

$$\int (x^2 - e^{2x} + \cos(3x))\, dx$$

(b) (8 points)

$$\int \frac{dx}{x \ln(2x)}$$

(c) (8 points)

$$\int_0^{\ln(\pi/4)} e^x \cos(e^x)\, dx$$

6. (20 pts)

Find the area between the curves

$$y = x^2, \quad y = x, \quad x = 0, \quad x = 2$$

7. (20 pts)

Find the volume of the solid obtained by rotating about the line $y = 2$, the region enclosed by the curves

$$y = \sqrt{x}$$

and

$$y = x.$$

8. (20 pts)

A spring has a natural length of $0.1m$. If a $20N$ force is required to keep it stretched to a length $0.3m$, how much work is required to stretch it from $0.1m$ to $0.2m$?

Part B

9. (16 pts) Solve the following integrals.

(a) (8 points)

$$\int x^2 e^{-2x}\, dx$$

(b) (8 points)

$$\int \frac{\ln x}{x^3}\, dx$$

10. (28 pts)

(a) (9 points) Find

$$\int \sin^2(x)\cos^3(x)\,dx$$

(b) (10 points) Find

$$\int \sin^2(x)\cos^2(x)\,dx$$

(c) (9 points) Find

$$\int \tan(x)\sec^3(x)\,dx$$

11. (18 pts) Solve the following integrals.

(a) (9 points)

$$\int \frac{x^3}{\sqrt{25-x^2}}\,dx$$

(b) (9 points)

$$\int \frac{dx}{x^2\sqrt{x^2+4}}$$

12. (36 pts) Solve the following integrals.

(a) (9 points)

$$\int \frac{2}{x^2 - x - 6}\, dx$$

(b) (9 points)

$$\int \frac{x^2 - x + 2}{x + 1}\, dx$$

(c) (9 points)

$$\int \frac{x+4}{x^3+2x^2}\,dx$$

(d) (9 points)

$$\int \frac{1}{x^3+3x^2+2x}\,dx$$

13. (8 pts) Approximate

$$\int_0^4 \sqrt{x^3+1}\,dx$$

using 4 intervals of equal length, using the trapezoidal rule. You do not have to evaluate the square roots.

14. (16 pts) Solve the following integrals.

(a) (8 points)

$$\int_{-1}^{4} \frac{1}{(x-1)^4}\, dx$$

(b) (8 points)

$$\int_{-3}^{-1} \frac{1}{(x+2)^{1/3}}\, dx$$

15. (8 pts) For the following problem, **SET UP THE INTEGRAL, BUT DO NOT SOLVE IT**. What is the length of the curve $y = x^3 + x$ between $x = -1$ and $x = 3$?

16. (10 pts) Suppose that a cubical tank with 2 meters on each side is full of water. Find the force on one of the vertical sides, in Newtons.

17. (10 pts) Find the center of mass of a right triangle with vertices at $(0, 0)$, $(0, 1)$, and $(1, 0)$.