

# MATH 142

## FINAL EXAM

May 5, 2003

NAME (please print legibly): \_\_\_\_\_

Your University ID Number: \_\_\_\_\_

Circle your Instructor's Name along with the Lecture Time:

Caulk (9 o'clock)      Knightly (10 o'clock)      Moustafaev (2 o'clock)      Qiu (2 o'clock)

- No calculators are allowed on this exam.
- Everyone must take part 1- it counts 10% of your grade
- In addition, your score on part 1 will replace your lowest midterm score if your part 1 score is higher.
- Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.

| Part I   |       |       |
|----------|-------|-------|
| QUESTION | VALUE | SCORE |
| 1        | 12    |       |
| 2        | 25    |       |
| 3        | 12    |       |
| 4        | 12    |       |
| 5        | 12    |       |
| 6        | 12    |       |
| 7        | 15    |       |
| TOTAL    | 100   |       |

| Part II  |       |       |
|----------|-------|-------|
| QUESTION | VALUE | SCORE |
| 1        | 10    |       |
| 2        | 12    |       |
| 3        | 16    |       |
| 4        | 20    |       |
| 5        | 10    |       |
| 6        | 12    |       |
| 7        | 20    |       |
| TOTAL    | 100   |       |

**Part I**

1. (12 pts) An ant crawls along a straight line with velocity

$$v(t) = 4 - 2t.$$



(a) Find the displacement of the ant between  $t = 0$  and  $t = 10$ .

ANSWER: \_\_\_\_\_

(b) Find the total distance traveled by the ant between  $t = 0$  and  $t = 10$ .

ANSWER: \_\_\_\_\_

**2. (25 pts)** Let  $f(x) = x + \frac{1}{x}$ .

(a) Find the critical numbers of  $f$ .

(b) Find the intervals where  $f$  is increasing and decreasing.

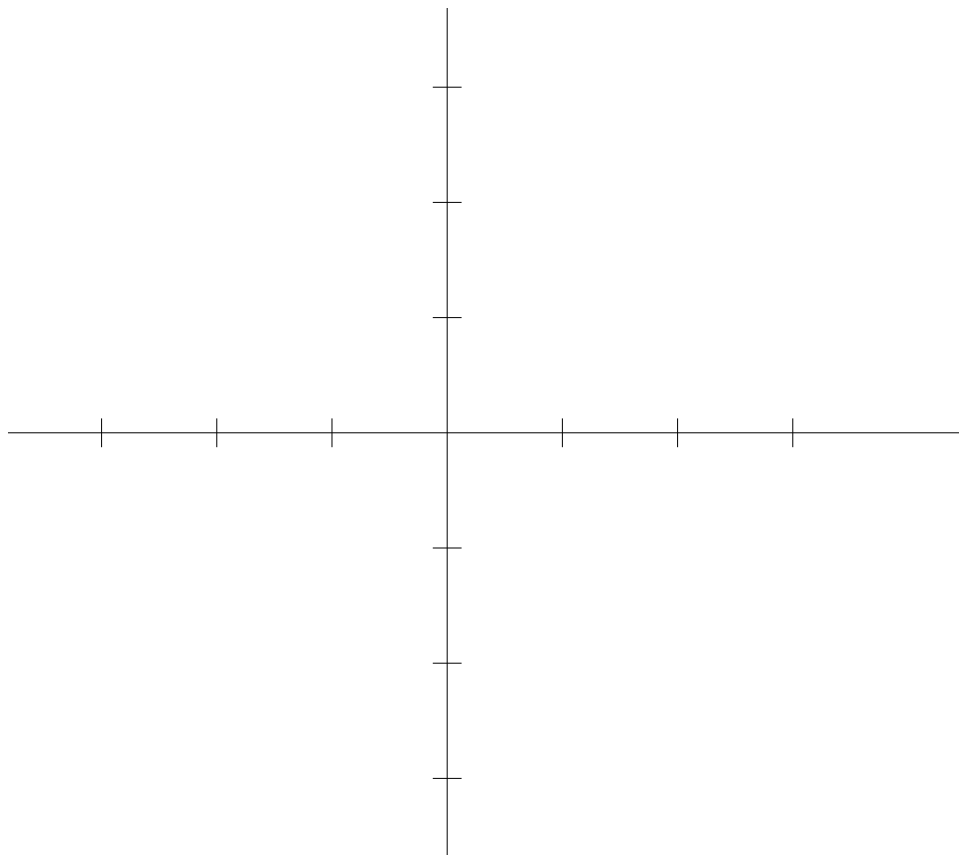
(c) Find all local extrema for  $f$ .

...Continuing with  $f(x) = x + \frac{1}{x}$ .

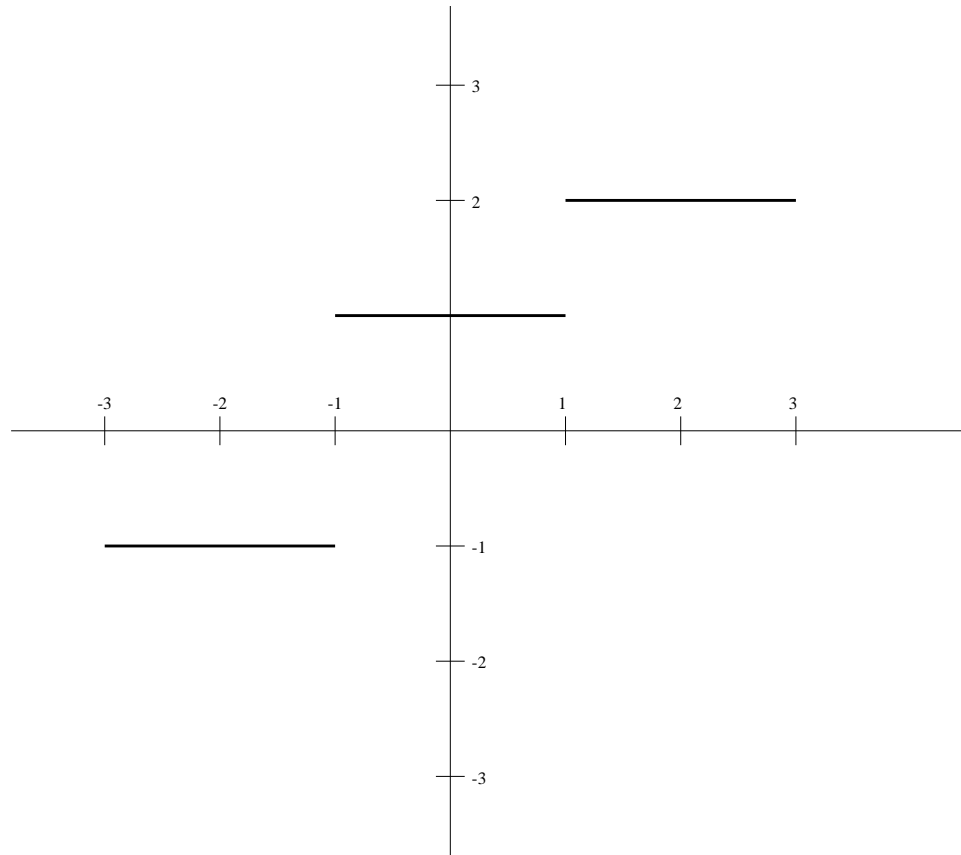
(d) Find the intervals where  $f$  is concave up/concave down.

(e) Show that  $f$  has a vertical asymptote at  $x = 0$ .

(f) Graph the curve  $y = f(x)$  on the given axes



3. (12 pts) Below is the graph of a (discontinuous) function  $y = f(x)$ .



Do the following (no need to show any work):

(a) Find  $\int_0^3 f(x)dx$ .

(b) Find  $\int_0^3 2f(x)dx$ .

(c) Find  $\int_{-3}^0 f(x)dx$ .

(d) Find  $\int_0^{-3} f(x)dx$ .

4. (12 pts) The difference of two numbers is 10. What is the smallest possible value for their product? (Show that your answer is the minimum).

5. (12 pts) Consider the region under the graph of  $y = \sqrt{x}$  between  $x = 0$  and  $x = 4$ . A solid is formed by rotating this region around the  $y$ -axis.

Draw a picture of the region:

- (a) Using  $dx$  to compute the volume will result in:    **Washers**    **Shells**    (circle one).
- (b) Using  $dy$  to compute the volume will result in:    **Washers**    **Shells**    (circle one).
- (c) Compute the volume of the solid, using whichever method you prefer.

**6. (12 pts)** A carpet which is 8 meters long is rolled up. When  $x$  meters have been unrolled, the force required to unroll it further is

$$F(x) = e^x(8 - x) \text{ Newtons.}$$

How much work does it take to unroll the entire carpet?





7. (15 pts) Compute the following integrals:

(a)  $\int x \sin(x^2) dx$

(b)  $\int_0^3 \frac{x^2}{\sqrt{x^3 + 1}} dx$

(c)  $\int x^2 \ln x dx$

**Part II**

**1. (10 pts)** Compute the following integrals.

(a)  $\int \cos^3(x) dx$

(b)  $\int \tan^2(x) \sec^4(x) dx$

2. (12 pts) An aphid crawled in the plane from  $(0, 0)$  to  $(4, \frac{16}{3})$  along the curve  $y = \frac{2}{3}x^{3/2}$ . How far did he crawl?



**3. (16 pts)** Compute the following integrals.

(a) (10 points)  $\int \frac{x + 3}{x^2 + 9} dx$

(b) (6 points)  $\int \frac{1}{e^x + 1} dx$

4. (20 pts) Compute the following integrals.

(a) (10 points)  $\int \frac{x^3}{\sqrt{x^2 + 4}} dx$

(b) (10 points)  $\int \frac{2x + 3}{x^2(x - 1)} dx$

5. (10 pts) An asteroid is moving in a straight line. Its velocity is measured at 2 second intervals. These measurements are tabulated below:

| Time | km/sec |
|------|--------|
| 0    | 5.1    |
| 2    | 4.1    |
| 4    | 3.5    |
| 6    | 2.0    |
| 8    | 1.2    |



Use Simpson's rule to estimate the distance traveled by the asteroid between  $t = 0$  and  $t = 8$ . You don't need to simplify.

**6. (12 pts)** Evaluate the following integrals. Show your work.

(a)  $\int_2^{\infty} \frac{1}{x^2} dx$

ANSWER: \_\_\_\_\_

(b)  $\int_0^2 \frac{1}{x^2} dx$

ANSWER: \_\_\_\_\_



**7. (20 pts)** A flat plastic plate has the shape of the region under the graph of  $y = \sin x$  between  $x = 0$  and  $x = \pi$ .

(a) Find the area of the plate.

(b) Find the  $x$ -coordinate of the center of mass of the plate.

(c) Find the  $y$ -coordinate of the center of mass of the plate.