MTH142

Final Exam

December 16, 2005

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Circle your instructor's name and lecture time.

Ethan Pribble	MWF 9:00 - 9:50 AM
Micah Milinovich	MW 3:25 - 4:40 PM

- **No calculators are allowed on this exam.**
- **Write legibly. Label sketches. Label and circle your answers.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. Do not include irrelevant work.**
- **Problems are not ordered according to difficulty. We recommend looking at all problems first and then starting with the ones that seem easiest to you.**

Part A		
QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
TOTAL	50	

Part B		
QUESTION	VALUE	SCORE
1	20	
2	10	
3	15	
4	15	
5	10	
6	10	
7	20	
TOTAL	100	

Part A

1. (10 points)

A model rocket is launched upward with an initial velocity of 48 feet per second off the edge of a platform located 64 feet above ground level. You may assume the acceleration due to gravity is constant at -32 feet per second squared.

(a) (3 points) Find the velocity function $v(t)$ which gives the velocity of the rocket in feet per second at time t in seconds after the launch.

(b) (3 points) Find the position function $s(t)$ which gives the height of the rocket in feet above the ground at time t in seconds after the launch.

(c) (2 points) How high does the rocket go?

(d) (2 points) When does the rocket hit the ground?

2. (10 points)

Evaluate the following derivatives.

(a) (2 points) $\frac{d}{dx}\int_2^x \cos t\,dt$

(b) (3 points) $\frac{d}{dx}\int_x^{x^3} \ln t\,dt$

Evaluate the following definite integrals.

(a) (2 points) $\int_0^2 x^3\,dx$

(b) (3 points) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$

3. (10 points)

Evaluate the following indefinite integrals using a substitution.

(a) (3 points) $\int e^{\tan x} \sec^2 x \, dx$

(b) (3 points) $\int \frac{1}{x \ln x} \, dx$

(c) (4 points) $\int x^7 \sqrt{x^4 + 8} \, dx$

4. (10 points)

Let S be the solid obtained by rotating the planar region bounded by the curves $y = x$ and $y = x^2$ about the line $y = 2$.

(a) (5 points) Write a definite integral which represents the volume of S using the definition of volume. Do NOT evaluate the definite integral.

(b) (5 points) Write a definite integral which represents the volume of S using the method of cylindrical shells. Do NOT evaluate the definite integral.

5. (10 points)

Evaluate the following indefinite integrals using integration by parts.

(a) (3 points) $\int xe^x \, dx$

(b) (3 points) $\int \ln x \, dx$

(c) (4 points) $\int x^2 \sin x \, dx$

Part B

1. (20 points)

(a) (5 points) Use the substitution $u = \sin x$ to evaluate the indefinite integral

$$\int \sin^6 x \cos^3 x \, dx.$$

(b) (5 points) Use the substitution $u = \sec x$ to evaluate the indefinite integral

$$\int \sec^6 x \tan^5 x \, dx.$$

(c) (5 points) Use a trigonometric identity to evaluate the indefinite integral

$$\int \cos^4 x \, dx.$$

(d) (5 points) Use a trigonometric identity to evaluate the indefinite integral

$$\int \sin(3x) \sin(2x) \, dx.$$

2. (10 points)

(a) (5 points) Use the substitution $x = 5\tan\theta$ to evaluate the indefinite integral

$$\int \frac{1}{\sqrt{x^2 + 25}}\,dx.$$

(b) (5 points) Complete the square to evaluate the indefinite integral

$$\int \frac{1}{\sqrt{-8 + 6x - x^2}}\,dx.$$

3. (15 points)

(a) (5 points) Rewrite the following rational function as a sum of a polynomial and a proper rational function.

$$\frac{x^5 + x^4 + 2x^2 + x + 1}{x^3 + 1}$$

(b) (5 points) The partial fraction decomposition of a certain proper rational function is

$$\frac{x^3 + 3x^2 + 1}{x^4 + x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}.$$

Find the values of the constants A, B, C and D.

(c) (5 points) Write the general form of the partial fraction decomposition of the proper rational function

$$\frac{x^2+2}{x^3(x^2-1)(x^2+4)^2}.$$

Your answer should include unresolved constants (A, B, C, and so on). Do NOT solve for the values of these constants.

4. (15 points)

Compute each of the following approximations to the definite integral

$$\int_0^{12} \sin\left(x^2\right)\, dx.$$

(a) (5 points) The 6th midpoint approximation M_6

(b) (5 points) The 6th trapezoid approximation T_6

(c) (5 points) The 6th Simpson approximation S_6

5. (10 points) For each of the following improper integrals state whether or not the integral converges or diverges. If the integral converges, find its value. If the integral diverges, state whether it diverges to $+\infty$, $-\infty$ or neither.

(a) (5 points) $\int_1^\infty \frac{1}{x^3} dx$

(b) (5 points) $\int_0^1 \frac{1}{x^3} dx$

6. (10 points)

Find the length of the curve $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}$ from $x = 1$ to $x = 2$.

7. (20 points)

Evaluate the following indefinite integrals.

(a) (5 points) $\displaystyle\int \frac{2}{x^2 - 1}\, dx$

(b) (5 points) $\displaystyle\int e^x \cos x \, dx$

(c) (5 points) $\displaystyle\int \frac{e^x}{e^{2x} + 2e^x + 1}\,dx$

(d) (5 points) $\displaystyle\int \arctan x\,dx$

Pythagorean Theorem identities

- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$

Double-angle identities

- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$

Half-angle identities

- $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- $\sin^2 x = \dfrac{1 - \cos 2x}{2}$

Product identities

- $\sin A \cos B = \dfrac{\sin(A-B) + \sin(A+B)}{2}$
- $\sin A \sin B = \dfrac{\cos(A-B) - \cos(A+B)}{2}$
- $\cos A \cos B = \dfrac{\cos(A-B) + \cos(A+B)}{2}$

Some indefinite integrals

- $\displaystyle\int \tan x \, dx = \ln|\sec x| + C$
- $\displaystyle\int \sec x \, dx = \ln|\sec x + \tan x| + C$