# MTH142

Midterm Exam 1

October 18, 2005

**NAME (please print legibly):** ______________________________

**Your University ID Number:** ______________________________

**Circle your instructor's name and lecture time.**

| | |
|---|---|
| Ethan Pribble | MWF 9:00 - 9:50 AM |
| Micah Milinovich | MW 3:25 - 4:40 PM |

- **No calculators are allowed on this exam.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Label and circle your answers.**
- **Problems are not ordered according to difficulty. We recommend looking at all problems first and then starting with the ones that seem easiest to you.**

| QUESTION | VALUE | SCORE |
|---|---|---|
| 1 | 16 | |
| 2 | 9 | |
| 3 | 6 | |
| 4 | 12 | |
| 5 | 12 | |
| 6 | 6 | |
| 7 | 6 | |
| 8 | 9 | |
| 9 | 9 | |
| 10 | 9 | |
| 11 | 6 | |
| TOTAL | 100 | |

**1. (16 points)**

Consider the function $f$ defined by

$$f(x) = \frac{-2(x^2-1)}{x^2-9}.$$

The first and second derivatives of $f$ are

$$f'(x) = \frac{32x}{(x^2-9)^2} \qquad \text{and} \qquad f''(x) = \frac{-96\,(x^2+3)}{(x^2-9)^3}.$$

(a) What is the domain of $f$?

(b) What are the $x$-intercept(s) of the graph of $f$? What are the $y$-intercept(s) of the graph of $f$?

(c) Find all horizontal asymptotes to the graph of $f$, if any exists, using the definition of horizontal asymptote.

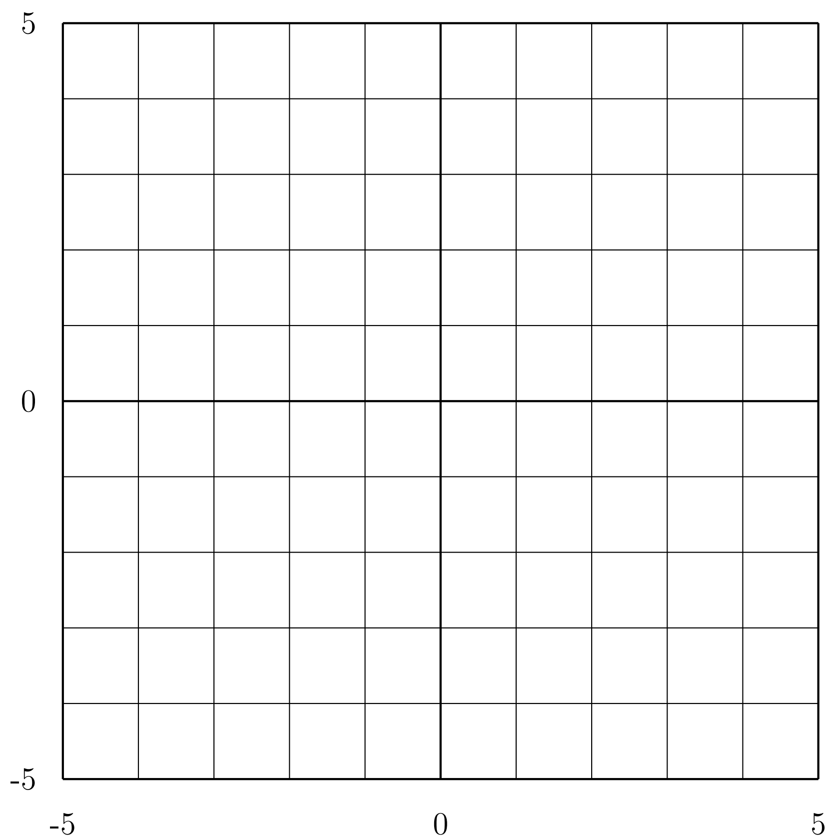
(d) Find all vertical asymptotes to the graph of $f$, if any exists, using the definition of vertical asymptote.

(e) Where is $f$ increasing? Where is $f$ decreasing?

(f) The function $f$ has one critical point. Identify that point and determine if $f$ has a local maximum, a local minimum, or neither at that point.

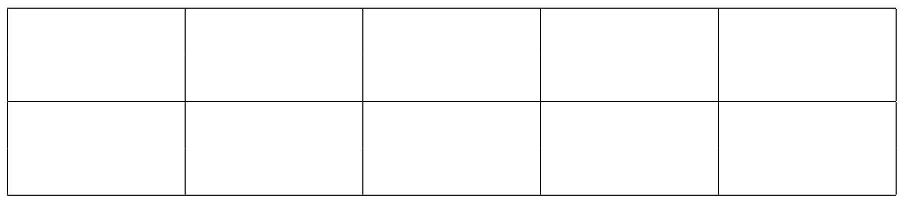
(g) Where is the graph of $f$ concave up? Where is the graph of $f$ concave down?

(h) Sketch the graph of $f$.



**2. (9 points)**

A farmer with 72 meters of fencing wants to fence off 10 small rectangular fields by building 3 horizontal fences and 6 vertical fences as shown in the following diagram.



What are the dimensions of the large rectangular field which will yield the largest area after being subdivided into 10 smaller rectangular regions and fenced off with 72 meters of fencing as described above? Explain why your answer yields the the absolute maximum.

**3. (6 points)**

(a) The equation $x^3+x+3=0$ has exactly one real solution between $-2$ and $-1$. Use Newton's Method with initial approximation $x_1 = -1$ to find $x_2$, the second approximation to the solution of the equation $x^3+x+3=0$.

(b) Explain how to use Newton's Method to approximate $\sqrt{2}$ correct to eight decimal places. Do NOT compute an approximation; simply explain how to use Newton's Method to obtain such an approximation.

**4. (12 points)**

A model rocket is launched upward with an initial velocity of 64 feet per second off the edge of a platform located 80 feet above ground level.

(a) Find the velocity function $v(t)$ which gives the velocity of the rocket in feet per second at time $t$ in seconds after the launch.

(b) Find the position function $s(t)$ which gives the height of the rocket in feet above the ground at time $t$ in seconds after the launch.

(c) How high does the rocket go?

(d) When does the rocket hit the ground?

**5. (12 points)**

Consider the region $R$ under the curve $y = x^3$ from $x = 0$ to $x = 4$.

(a) Write the Riemann sum which estimates the area of $R$ using four rectangles of equal width and left endpoints. Is this sum an underestimate or an overestimate?

(b) Write the Riemann sum which estimates the area of $R$ using four rectangles of equal width and right endpoints. Is this sum an underestimate or an overestimate?

(c) Express the area of $R$ as a limit of Riemann sums.

(d) Compute the area of $R$ using any valid method. You may want to use the following formula.

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

**6. (6 points)**

Evaluate the following definite integrals using geometry.

(a) $\displaystyle\int_0^3 \sqrt{9 - x^2}\, dx$

(b) $\displaystyle\int_{-1}^3 |x|\, dx$

**7. (6 points)**

Suppose $f$ is a function satisfying

$$\int_{-1}^{7} f(x)\,dx = 4 \qquad \text{and} \qquad \int_{6}^{7} f(x)\,dx = 12.$$

Evaluate the following integrals.

(a) $\displaystyle\int_{-1}^{6} f(x)\,dx$

(b) $\displaystyle\int_{-1}^{7} (7f(x) - 3)\ dx$

**8. (9 points)**

Evaluate the following derivatives.

(a) $\dfrac{d}{dx}\displaystyle\int_7^x e^{t^3}\,dt$

(b) $\dfrac{d}{dx}\displaystyle\int_2^{x^7} \cos t\,dt$

(c) $\dfrac{d}{dx}\displaystyle\int_x^{x^2} \sin t\,dt$

**9. (9 points)**

Evaluate the following definite integrals.

(a) $\displaystyle\int_0^{\pi} \sin x \, dx$

(b) $\displaystyle\int_1^{2} \frac{1}{x} \, dx$

(c) $\displaystyle\int_0^{1} e^x \, dx$

**10. (9 points)**

Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{2x^3 - x}{\sqrt{x}}\, dx$

(b) $\displaystyle\int \sec x \tan x \, dx$

(c) $\displaystyle\int \frac{7}{x^2 + 1}\, dx$

**11. (6 points)**

A particle travels in a straight line with velocity $v(t) = t^2 - 4t + 3$ meters per second.

(a) Find the displacement of the particle from time $t = 0$ to time $t = 2$.

(b) Find the total distance travelled by the particle from time $t = 0$ to time $t = 2$.