MTH 142 **Midterm 2** **November 17, 2016**

Last/Family Name: ______________________________

First/Given Name: ______________________________

Student ID Number: ______________________________

Instructor (circle): Hambrook (MW 3:25) Zeng (MW 9:00)

Honor Pledge: "I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

You must write out and sign the honor pledge for your examination to be valid.

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Signature: ______________________________ Date: ______________________________

Instructions:

Solutions

- Time: 75 minutes.
- Write in pencil or pen.
- No notes, textbooks, phones, calculators, or other electronic devices.
- If you need extra space, use the back of the page, and indicate it.
- To receive full credit, you must show your work and justify your answers.
- The final page is a formula sheet. You may detach it.

QUESTION	VALUE	SCORE
1	14	
2	15	
3	17	
4	17	
5	12	
TOTAL	75	

1. (14 points) Evaluate the integrals.

(a) $\int x^5 \ln x \, dx$

Use integration by parts

$u = \ln x \quad dv = x^5 dx$

$du = \frac{1}{x} dx \quad v = \frac{1}{6} x^6$

$= \frac{1}{6} \ln x \cdot x^6 - \int \frac{1}{6} x^6 \cdot \frac{1}{x} dx$

$= \frac{x^6 \ln x}{6} - \frac{1}{6} \int x^5 dx$

$= \frac{x^6 \ln x}{6} - \frac{1}{36} x^6 + C.$

(b) $\int x^5 \sqrt{1+x^3} \, dx$

Use subsitution

$u = 1 + x^3$

$du = 3x^2 dx$

$dx = \frac{1}{3x^2} du.$

$= \int x^5 \sqrt{u} \cdot \frac{1}{3x^2} du$

$= \frac{1}{3} \int x^3 \sqrt{u} \, du$

$= \frac{1}{3} \int (u-1) \sqrt{u} \, du = \frac{1}{3} \int (u^{\frac{3}{2}} - u^{\frac{1}{2}}) du$

$= \frac{1}{3} \left(\frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} \right) + C$

$= \frac{1}{3} \left[\frac{2}{5} (1+x^3)^{\frac{5}{2}} - \frac{2}{3} (1+x^3)^{\frac{3}{2}} \right] + C$

$= \frac{2}{15} (1+x^3)^{\frac{5}{2}} - \frac{2}{9} (1+x^3)^{\frac{3}{2}} + C$

2. (15 points) Evaluate the integrals.

(a) $\int_0^{\pi/4} \sec^4 x\, dx$

Use subsitution

$u = \tan x$.

$du = \sec^2 x\, dx$

$$= \int_0^1 \sec^4 x \cdot \frac{1}{\sec^2 x}\, du$$

$$= \int_0^1 \sec^2 x\, du = \int_0^1 (\tan^2 x + 1)\, du = \int_0^1 (u^2+1)\, du$$

$$= \frac{1}{3}u^3 + u \Big|_0^1$$

$$= \frac{4}{3}$$

(b) $\int x \cos^2 x\, dx$

$$= \int x \cdot \frac{1+\cos(2x)}{2}\, dx = \int \frac{x}{2}\, dx + \frac{1}{2}\int x\cos(2x)\, dx$$

For the second integral. use. integration by parts.

$u = x$. $dv = \cos(2x)\, dx$

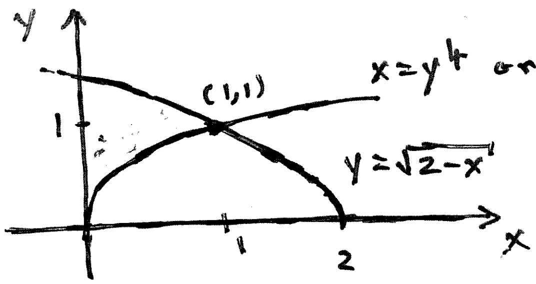
$du = dx$ $v = \frac{1}{2}\sin(2x)$

$$\int x\cos(2x)\, dx = \frac{x}{2}\sin(2x) - \int \frac{1}{2}\sin(2x)\, dx = \frac{x\sin(2x)}{2} + \frac{1}{4}\cos(2x) + C$$

Thus original problem $= \frac{1}{4}x^2 + \frac{x\sin(2x)}{4} + \frac{1}{8}\cos(2x) + C$

3. (17 points) Sketch the region enclosed by the given curves and find its area.

$$x = y^4, \quad y = \sqrt{2-x}, \quad y = 0.$$



$x = y^4$ or $y = x^{1/4}$ $(y \geq 0)$

$y = \sqrt{2-x}$ or $x = 2 - y^2$ $(x \geq 0)$

Intersection Point

$x = y^4$, $y = \sqrt{2-x}$

$x = y^4 = (\sqrt{2-x})^4$

$x = (2-x)^2$

$x = x^2 - 4x + 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 1$ or $\cancel{x=4}$

$y = \sqrt{2-1} = 1$ not in domain

Solution 1 (integrate in x)

$$\text{Area} = \int_0^1 (x^{1/4} - 0)\,dx + \int_1^2 (\sqrt{2-x} - 0)\,dx$$

$u = 2 - x$ $x = 1 \Rightarrow u = 1$

$du = -dx$ $x = 2 \Rightarrow u = 0$

$$= \frac{4}{5} x^{5/4} \Big|_0^1 - \int_1^0 u^{1/2}\,du$$

$$= \frac{4}{5} x^{5/4} \Big|_0^1 - \frac{2}{3} u^{3/2} \Big|_1^0$$

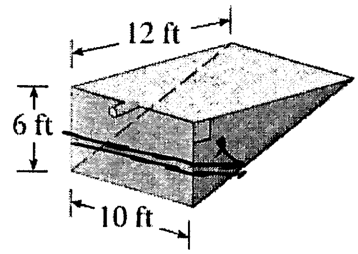
$$= \frac{4}{5} + \frac{2}{3} = \frac{22}{15}$$

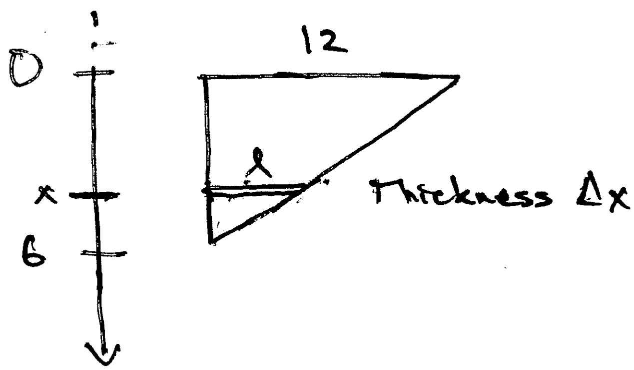
Solution 2 (integrate in y)

$$\text{Area} = \int_0^1 (2 - y^2 - y^4)\,dy = 2y - \frac{1}{3}y^3 - \frac{1}{5}y^5 \Big|_0^1$$

$$= 2 - \frac{1}{3} - \frac{1}{5} = \frac{22}{15}$$

4. (17 points) The tank pictured is full of water. Find the work required to pump the water out of the spout at the top. Use that water weighs 62.5 lb/ft^3.





$$\frac{l}{12} = \frac{6-x}{6}$$

$$l = 12 - 2x$$

Volume of slab $\approx 10 l \Delta x = 10(12-2x)\Delta x = 20(6-x)\Delta x$

mass of slab $\approx (62.5)(20)(6-x)\Delta x$

force on slab $\approx (62.5)(20)(6-x)\Delta x$ (1 lb mass feels 1 lb force due to gravity)

distance slab moves $= x$

work on slab $=$ force $\cdot$ distance $\approx (62.5)(20)(6-x)\,x\,\Delta x$

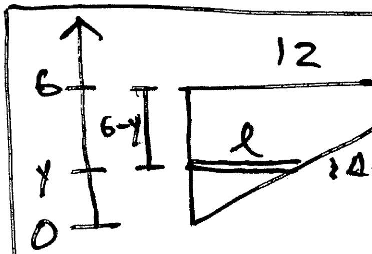
$$W = \int_0^6 (62.5)(20)(6-x)x\,dx$$

$$= (62.5)(20)\int_0^6 (6x - x^2)\,dx$$

$$= (62.5)(20)\left(3x^2 - \tfrac{1}{3}x^3\right)\Big|_0^6$$

$$= (62.5)(20)\left(3\cdot 6^2 - \tfrac{1}{3}6^3\right)$$

$$= 45000 \text{ (ft}\cdot\text{lb)}$$



$$\frac{l}{12} = \frac{y}{6}$$

$$l = 2y$$

volume $\approx 10 l \Delta y = 20 y \Delta y$

force $\approx (62.5)(20) y \Delta y$

distance moved $= 6-y$

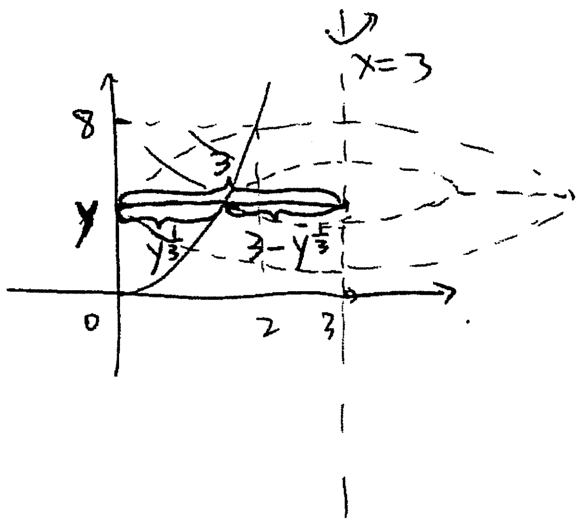
$$W = \int_0^6 (62.5)(20)(6-y)y\,dy$$

$=$ same

5. (12 points) Consider the solid obtained when the region bounded by the curves

$$y = x^3, \quad y = 8, \quad x = 0$$

is rotated about the line $x = 3$. Use the method of **disks/washers** to set up an integral equal to the volume of the solid. Do **not** evaluate the integral.



$$V = \int_0^8 \pi \left[3^2 - (3 - y^{\frac{1}{3}})^2 \right] dy$$

Formula Sheet

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\text{arcsec}\, x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\text{arccsc}\, x = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\text{arccot}\, x = -\frac{1}{1+x^2}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin^2 x = \frac{1}{2}(1-\cos(2x)) \qquad \sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

$$\cos^2 x = \frac{1}{2}(1+\cos(2x)) \qquad \cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$

$$\sin x \cos x = \frac{1}{2}\sin(2x) \qquad \sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \sec x \, dx = \ln|\sec x + \tan x| + C$$