MTH 142 **Practice Final Exam** **December 17, 2016**

Last/Family Name: ______________________________

First/Given Name: ______________________________

Student ID Number: ______________________________

Instructor (circle): Hambrook (MW 3:25) Zeng (MW 9:00)

Honor Pledge: "I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

You must write out and sign the honor pledge for your examination to be valid.

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Signature: ______________________ Date: ______________________

QUESTION	VALUE	SCORE
1	15	
2	10	
3	15	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	15	
13	10	
14	10	
15	10	
TOTAL	165	

Instructions:

- Time: 3 hours.
- Write in pencil or pen.
- No notes, textbooks, phones, calculators, or other electronic devices.
- If you need extra space, use the back of the page, and indicate it.
- To receive full credit, you must show your work and justify your answers.
- The final page is a formula sheet. You may detach it.

1. (15 points) Consider the function f with

$$f(x) = \frac{x^2 - 4}{x^2 + 2x + 1}, \quad f'(x) = \frac{2(x+4)}{(x+1)^3}, \quad f''(x) = \frac{-2(2x+11)}{(x+1)^4}.$$

(a) Find the domain of f.

(b) List all x- and y-intercepts of f.

(c) List all vertical asymptotes of f or explain why none exist.

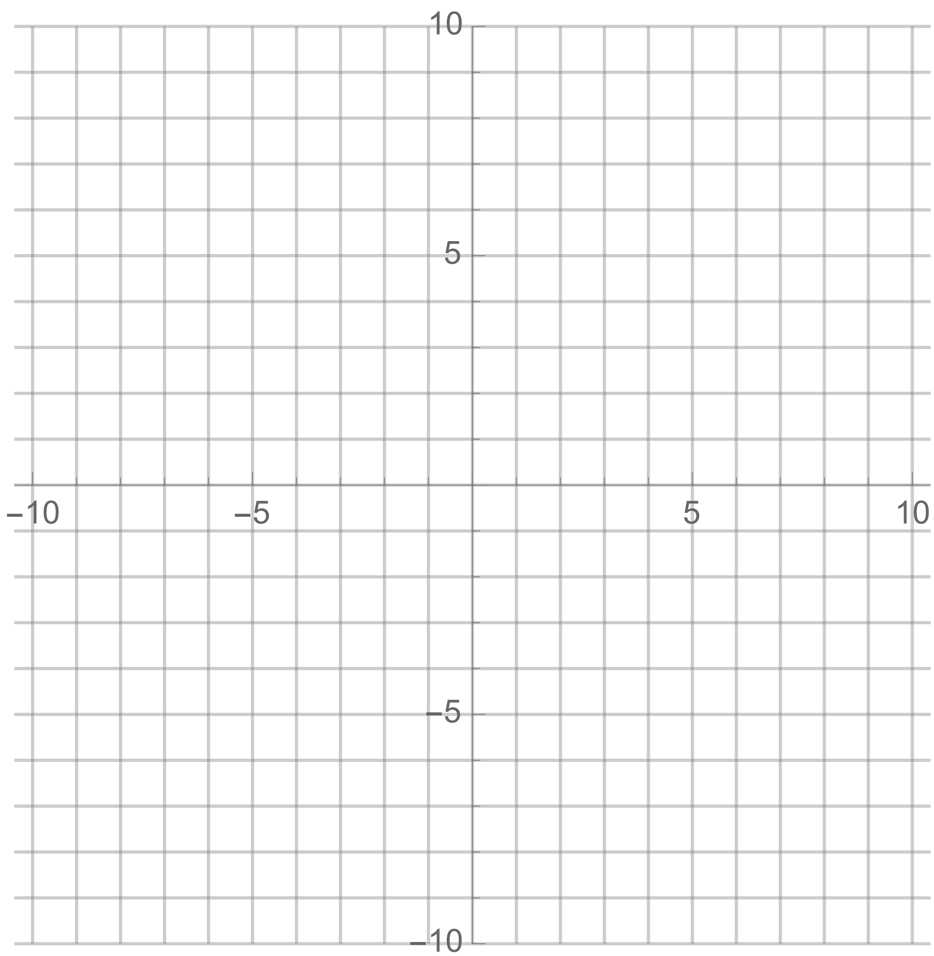
(d) List all horizontal asymptotes of f or explain why none exist.

(e) On what intervals is $f(x)$ increasing? decreasing?

(f) On what intervals is $f(x)$ concave up? concave down?

2. (10 points) Sketch the graph of a function $g(x)$ that satisfies the following properties:

- g is continuous at all points of its domain
- x-intercepts: $-5, -1, 1$
- $\lim_{x\to 0^-} g(x) = -\infty$ and $\lim_{x\to 0^+} g(x) = +\infty$
- $\lim_{x\to -\infty} g(x) = -2$ and $\lim_{x\to +\infty} g(x) = -2$
- increasing on $(-\infty, -3)$
- decreasing on $(-3, 0) \cup (0, \infty)$
- concave up on $(-\infty, -4) \cup (0, \infty)$
- concave down on $(-4, 0)$



3. (15 points) Consider a line in the xy-plane that passes through the point $(8/9, 3)$ and has negative slope. It forms a right triangle with the x-axis and the y-axis. What is the smallest possible length of the hypotenuse?

4. (10 points)

The table gives the values of a function f obtained from an experiment.

x	3	5	7	9	11
$f(x)$	-3.4	-0.6	0.9	1.8	3.3

(a) Use the table to approximate $\int_3^{11} f(x)dx$ by a Riemann sum with left endpoints and four equal subintervals.

(b) If f is known to be an increasing function, is your estimate larger or smaller than the exact value of the integral.

5. (10 points) Evaluate:

(a) $\dfrac{d}{dt}\displaystyle\int_0^{\ln t} e^{-x^2}\,dx$

(b) $\displaystyle\int_0^{\ln t} \dfrac{d}{dx} e^{-x^2}\,dx$

6. (10 points) The velocity function (in meters per second) for a particle moving in one dimension is

$$v(t) = t^2 - t - 2$$

Find the total distance traveled during the time interval $1 \le t \le 4$.

7. (10 points) Evaluate the integral.

$$\int_0^{\pi/3} \sin^3 x \cos^3 x \, dx$$

8. (10 points) Evaluate the integral.

$$\int_0^1 \frac{1}{(2x-1)^2} dx$$

9. (10 points) Evaluate the integral.

$$\int \frac{x}{(2x-1)(x-1)^2}dx$$

10. (10 points) Evaluate the integral.

$$\int \sqrt{3 + 2x - x^2}\, dx$$

11. (10 points) Evaluate the integral.

$$\int x^2 e^{-x} dx$$

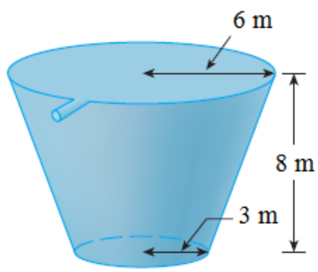
12. (15 points) Find the length of the curve

$$x = \frac{y^4}{8} + \frac{1}{4y^2}, \quad 1 \le y \le 2.$$

13. (10 points) Sketch the region enclosed by the given curves. Then set up an integral equal to its area. Do **not** evaluate the integral.

$$x = 2y^2, \quad x = 4 + y^2.$$

14. (10 points) The tank pictured is full of water. The density of water is 1000 kg/m^3. Set up an integral equal to the work required to pump the water out of the spout at the top. Do **not** evaluate the integral.



frustum of a cone

15. (10 points) The base of the solid is a circular disk with radius 2. Parallel cross-sections perpendicular to the base are squares. Set up an integral equal to the volume of the solid. Do **not** evaluate the integral.

Formula Sheet

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\operatorname{arccsc} x = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin^2 x = \frac{1}{2}(1-\cos(2x)) \qquad \sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

$$\cos^2 x = \frac{1}{2}(1+\cos(2x)) \qquad \cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$

$$\sin x \cos x = \frac{1}{2}\sin(2x) \qquad \sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$

$$\int \tan x\, dx = \ln|\sec x| + C \qquad \int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\, dx = -\ln|\csc x| + C \qquad \int \csc x\, dx = -\ln|\csc x + \cot x| + C$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$