

Math 142: Calculus II

Midterm 1 Practice

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the appropriate box:

Crossen	MW 9-10:15	<input type="checkbox"/>
Zhong	MW 3:25-4:40	<input type="checkbox"/>

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 9 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.
- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

QUESTION	VALUE	SCORE
1	15	
2	10	
3	10	
4	10	
5	15	
6	15	
TOTAL	75	

1. (15 points) Consider the following function with first and second derivative also given

$$f(x) = xe^{-x^2}, \quad f'(x) = (1 - 2x^2)e^{-x^2}, \quad f''(x) = 2x(2x^2 - 3)e^{-x^2}.$$

(a) Find the domain of f .

(b) List all x and y intercepts of f .

(c) Find all horizontal and vertical asymptotes of f or explain why none exist.

(d) Find all critical points and where the function is increasing and where it is decreasing.

(e) Find where the function is concave up, and where it is concave down, and all x values at points of inflection.

(f) Use the information you found in parts (a)-(e) to sketch the graph of $y = f(x)$.

2. (10 points) The Coca-Cola company wants to design a new, larger soda can in the shape of a perfect cylinder. Due to a shortage of aluminum, they will make the top and bottom of the can out of stainless steel, which costs 2 cents per square inch of surface area, and they will make the curved part of the can out of tin, which costs 1 cent per square inch of surface area. The volume of the can must be exactly 32π cubic inches (about 22 fluid ounces). Find the radius and the height of the can that minimizes the metal costs.

3. (10 points) A particle moves along the x -axis with position $s(t)$, velocity $v(t)$ and acceleration $a(t)$ for $t \geq 0$. Suppose that $a(t) = 12t - 42$, $s(0) = -3$, and $v(0) = 36$.

(a) Compute $v(t)$.

(b) Compute $s(t)$.

(c) On what time interval(s) is the particle moving forwards?

(d) What is the **distance** (not the displacement!) the particle travels during the interval $0 \leq t \leq 10$? (Note: you don't need to simplify numerical expressions.)

4. (10 points) Evaluate the following definite integrals by interpreting it terms of areas.

(a) $\int_0^1 |2x - 1| dx$

(b) $\int_{-7}^3 \sqrt{25 - (x + 2)^2} dx.$

5. (15 points) Estimate the area between the graph of $y = x^3 + 1$ and x -axis from $x = 0$ to $x = 2$ using a right-endpoint Riemann sum with:

(a) 4 rectangles

(b) n rectangles (express your answer in terms of n)

(c) Write the area as a definite integral and use your answer in part (b) to determine the exact value of the area. **(No partial credit if you use other methods.)**

6. (15 points) Compute the following using the Fundamental Theorem of Calculus.

(a) $f'(x)$ if $f(x) = \int_{\ln x}^5 \sqrt{1-t^2} dt$.

(b) $\int_0^{1/2} \frac{1}{\sqrt{1-t^2}} dt$.

(c) $\int_1^4 (2\sqrt{x}-1)(\sqrt{x}+2) dx$.

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