MATH 142 Post-Midterm 2 Review Sheet

- **Integration By Parts:** Differentiate one function, integrate the other, arrive at an easier integral.

Formula: $\int u\; dv = uv - \int v\; du$

ALSO REMEMBER: You might have to apply the formula more than once.

- **Trig Integral Tricks:** We discussed some tricks for integrating powers of trig functions, some of which are listed below:

- $\int \sin^n(x)\cos^m(x)dx$, if EITHER (or both) of n or m is odd: Whichever power is odd, borrow a factor of the corresponding trig function, and make a substitution with u equaling the OTHER trig function. Use trig identities when necessary to get everything in terms of u.

- In other situations, when dealing with even powers of sine and cosine, you may find the identities for $\sin^2(x)$ and $\cos^2(x)$ helpful. Recall that you will be given a list of trig identities, which are posted on the course webpage.

- $\int \tan^n(x)\sec^m(x)dx$, if $m \geq 2$ is even: Borrow $\sec^2(x)$, let $u = \tan x$, and use trig identities when necessary to get everything in terms of u.

- **Trig Substitution:** DRAW THE TRIANGLE

If You See	Maybe Try
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$
$\sqrt{x^2 + a^2}$	$x = a\tan\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$

- **Partial Fractions:** If you want to integrate a rational function (i.e. $\frac{p(x)}{q(x)}$ where p and q are polynomials)...

- If the degree of the numerator is greater than or equal to the degree of the numerator, you need to do polynomial long division first, which will leave you with $\frac{p(x)}{q(x)} = m(x) + \frac{r(x)}{q(x)}$, where $m(x)$ is a polynomial (and hence easy to integrate), and $\frac{r(x)}{q(x)}$ is a *proper* rational function, meaning the degree of the numerator is LESS than the degree of the denominator.

- Once you have a proper rational function, factor the denominator as much as possible, until you have it written as a product of powers of linear factors and powers of irreducible quadratic factors. For each power of each linear factor, you need a term in your decomposition with an unknown constant numerator.

For each power of each quadratic factor, you need a term in your decomposition with an unknown linear numerator. For example:

$$\frac{x^3+x-1}{x^3(x-4)(x^2+9)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-4} + \frac{Ex+F}{x^2+9} + \frac{Gx+H}{(x^2+9)^2}.$$

- Once you have your decomposition written out, multiply both sides of the equation by the entire denominator, which will in particular cancel all the denominators on the right side. Then, you need to solve for all the constants, which can be done by plugging in well-chosen values of x to both sides of the equation, by equating coefficients of the polynomials on each side of the equation, or by some combination of the two.

- Once you have all of your constants, you just integrate the decomposed function term by term, which may require other things like substitution or completing the square.

- Improper Integrals

Definitions:

$$\int_a^\infty f(x)\ dx = \lim_{b\to\infty}\int_a^b f(x)\ dx, \quad \int_{-\infty}^b f(x)\ dx = \lim_{a\to-\infty}\int_a^b f(x)\ dx$$

$$\int_{-\infty}^\infty f(x)\ dx = \int_{-\infty}^0 f(x)\ dx + \int_0^\infty f(x)\ dx.$$

If f is continuous on $[a,b)$ but discontinuous at $x=b$,

$$\int_a^b f(x)\ dx = \lim_{R\to b^-}\int_a^R f(x)\ dx,$$

and we have the analogous definition for discontinuity at $x=a$.

If the limit in question exists, we say the improper integral *converges*, otherwise we say it *diverges*.

***p*-integral Test:** For any $a>0$, $\int_a^\infty \frac{1}{x^p}\ dx$ converges if and only if $p>1$, and $\int_0^a \frac{1}{x^p}\ dx$ converges if and only if $p<1$.

Comparison Test: Suppose $0 \le g(x) \le f(x)$ for all $x \ge a$.

If $\int_a^\infty f(x)\ dx$ converges, then $\int_a^\infty g(x)\ dx$ converges.

If $\int_a^\infty g(x)\ dx$ diverges, then $\int_a^\infty f(x)\ dx$ diverges.

(Same for integrals around an asymptote.)

Note that you cannot conclude anything about $\int_a^\infty f(x)\,dx$ (the bigger one) if all you know is that $\int_a^\infty g(x)\,dx$ (the smaller one) converges. Similarly, you cannot conclude anything about $\int_a^\infty g(x)\,dx$ if all you know is that $\int_a^\infty f(x)\,dx$ diverges.

- **Arclength:** The length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$\text{arclength} = \int_a^b \sqrt{1 + f'(x)^2}\,dx$$