MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

OFF HRS: M, F 4-5PM ;
BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

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COURSE PAGE: bit.ly/sahay142

ANNOUNCEMENTS

1. WEBWORK DEADLINES : (F) WW 6 → FRIDAY, 11 PM
   (T) WW 7 → TUESDAY, 11 PM
   (TO BE RELEASED TONIGHT)

2. OFFICE HOURS TOMORROW (4-5 PM ET)

CONTD FROM LAST TIME

EXAMPLE 7 Figure 12 shows a solid with a circular base of radius 1. Parallel cross-sections perpendicular to the base are equilateral triangles. Find the volume of the solid.

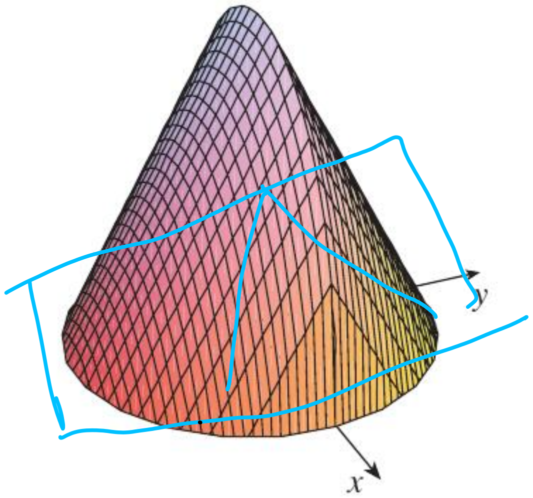
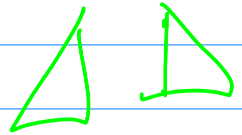
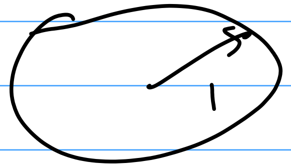
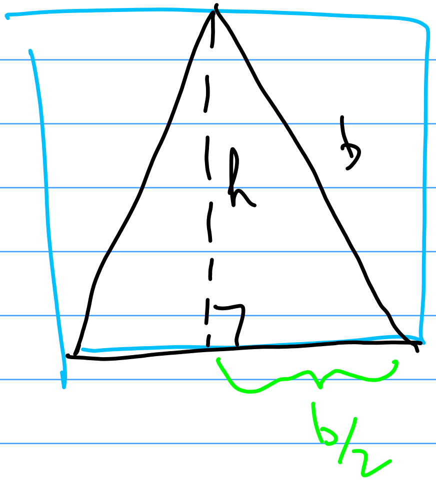


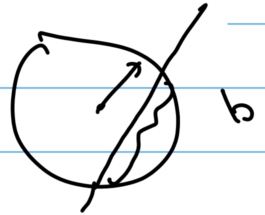
FIGURE 12



$$\boxed{h = \frac{b\sqrt{3}}{2}}$$







AREA OF $\Delta = \frac{1}{2} b h$

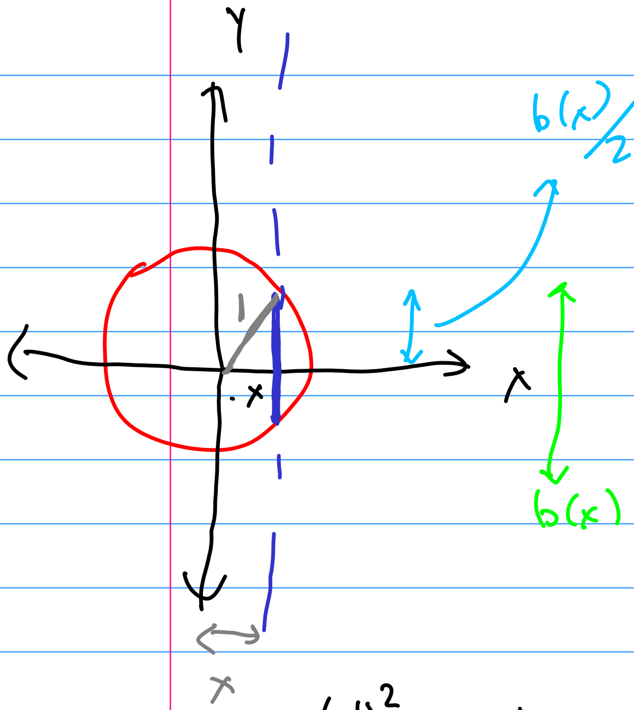
$$h^2 + \left(\frac{b}{2}\right)^2 = b^2 \Rightarrow h^2 = b^2 - \frac{b^2}{4} = \frac{3b^2}{4}$$

$$\text{AREA} = \frac{1}{2} b h \qquad \left(h = \frac{\sqrt{3}}{2} b\right)$$

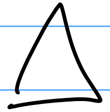
$$= \frac{1}{2} b \left(\frac{\sqrt{3}}{2} b\right)$$

$$= \frac{\sqrt{3}}{4} b^2$$

$$\boxed{\text{AREA OF EQUILATERAL } \Delta \text{ OF BASE } b = \frac{\sqrt{3}}{4} b^2}$$

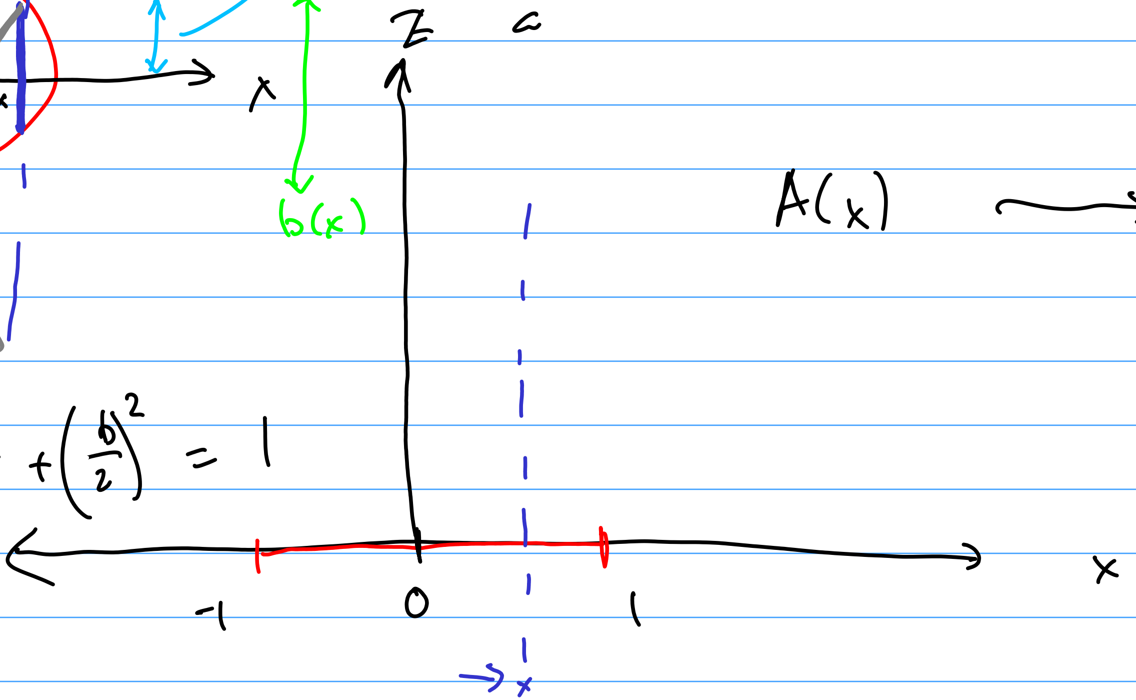


$$\int_a^b A(x) \, dx$$



$$A(x) \longrightarrow b(x)$$

PYTH: $x^2 + \left(\frac{b}{2}\right)^2 = 1$



$a = -1, \; b = 1$

$$x^2 + \frac{b^2}{4} = 1$$

$$\Rightarrow \quad b^2 = 4(1-x^2)$$

$A(x)$ = AREA OF AN ~~EQUILATERAL~~ Δ OF BASE b

$$= \frac{\sqrt{3}}{4} b^2 = \frac{\sqrt{3}}{4}\left[4(1-x^2)\right]$$

$$A(x) = \sqrt{3}\,(1-x^2)$$

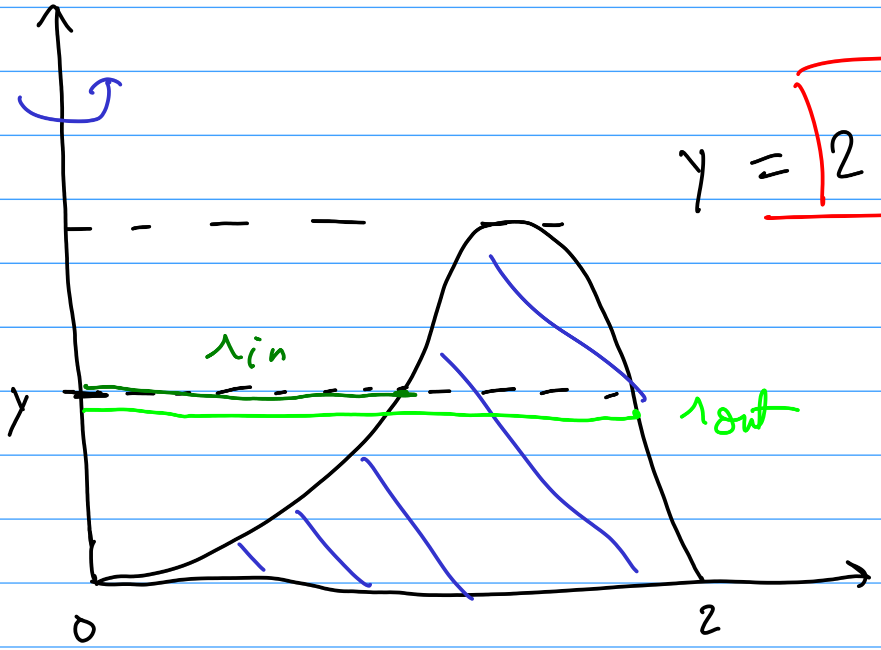
$$\int_{-1}^{1} A(x)\,dx = \int_{-1}^{1} \sqrt{3}\,(1-x^2)\,dx$$

$$= \sqrt{3}\left. x - \frac{x^3}{3} \right]_{-1}^{1}$$

$$= \left(\sqrt{3} - \frac{1}{3}\right) - \left(-\sqrt{3} + \frac{1}{3}\right)$$

$$= 2\sqrt{3} - 2/3$$

$\S$ 6.3 VOLUMES BY CYLINDRICAL SHELLS



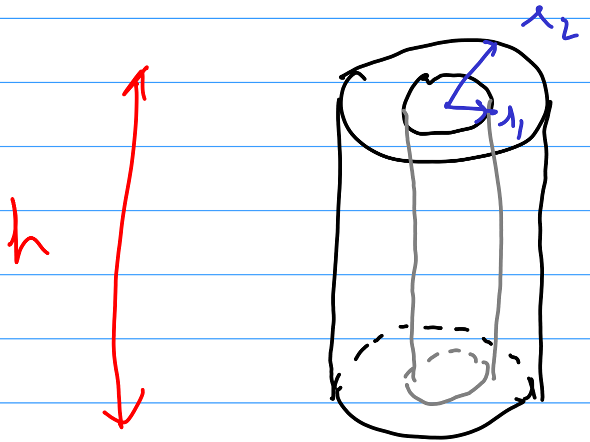
$$y = \boxed{2x^2 - x^3}$$

HARD TO INVERT.

FIND x, s.t. $y = 2x^2 - x^3$

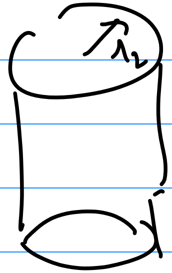
$$\int_a^b \pi \left[r_{out}^2 - r_{in}^2 \right] dy$$

SOLUTION? $\rightarrow$ APPROXIMATE BY CYLINDRICAL SHELLS.



VOLUME?

$$V = \pi r_2^2 h - \pi r_1^2 h = \pi h (r_2^2 - r_1^2)$$



$$V_2 = \pi r_2^2 h$$

$$V_1 = \pi r_1^2 h$$

$a^2 - b^2 = (a-b)(a+b)$

$$V = \pi h (r_2^2 - r_1^2)$$

$$= \pi h \underbrace{(r_2 - r_1)}(r_2 + r_1)$$

$$\pi h (\Delta r)(2r)$$

CIRCUMFERENCE

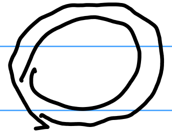
π

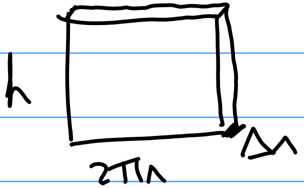
$2\pi r$

Δr → THICKNESS

$$= (2\pi r)\, h\, (\Delta r)$$

h = HEIGHT





$r_1 \approx r_2$

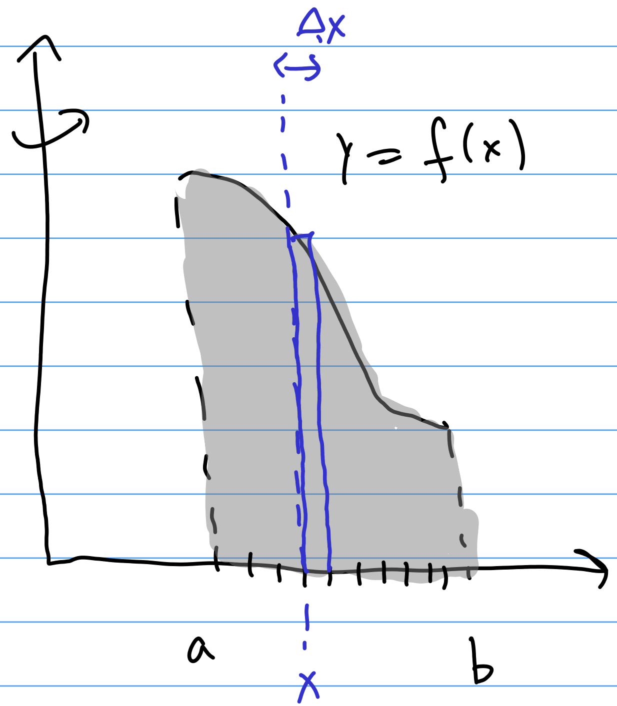
$\Delta r = (r_2 - r_1)$

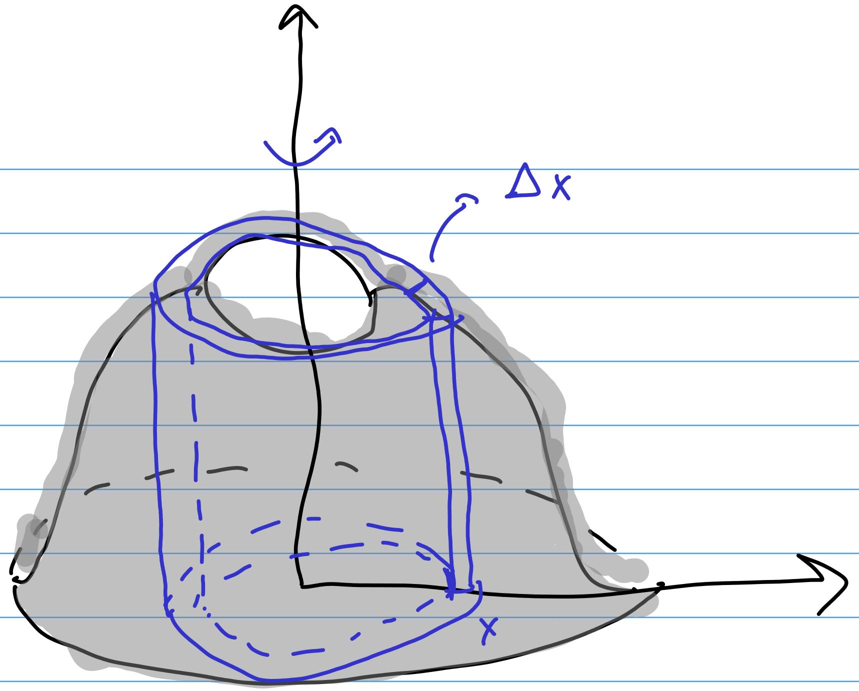
$\Delta r << r_1$

(r → MIDPT. OF r_1 & r_2)

$r_2 + r_1 = 2r$

VOLUME OF A CYLINDRICAL SHELL = CIRCUMFERENCE × HEIGHT × THICKNESS





$$\text{VOLUME OF SHELL AT } x = \overset{2\pi x}{[\text{CIRCUMFERENCE}]} \times \overset{f(x)}{[\text{HEIGHT}]} \times \overset{\Delta x}{[\text{THICKNESS}]} = 2\pi x\, f(x)\, \Delta x$$

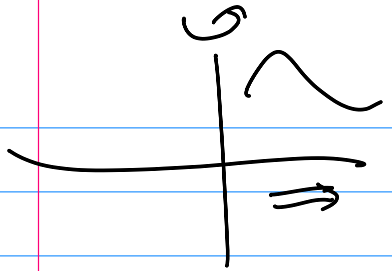
$$\sum_{j=1}^{n} 2\pi x_j f(x_j)\,\Delta x$$

$$\begin{pmatrix} x_1 = a \\ x_n = b \end{pmatrix}$$

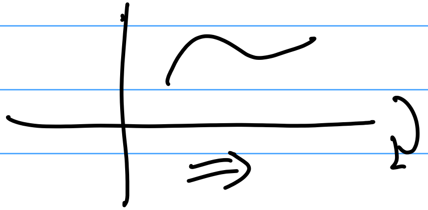
$$\xrightarrow[\Delta x \to \infty]{n \to \infty} \int_a^b \left[2\pi x\, f(x)\right] dx$$

$\therefore$ FORMULA FOR $y = f(x)$ b/w $x = a$ & $x = b$
ROTATED ABOUT THE y - AXIS

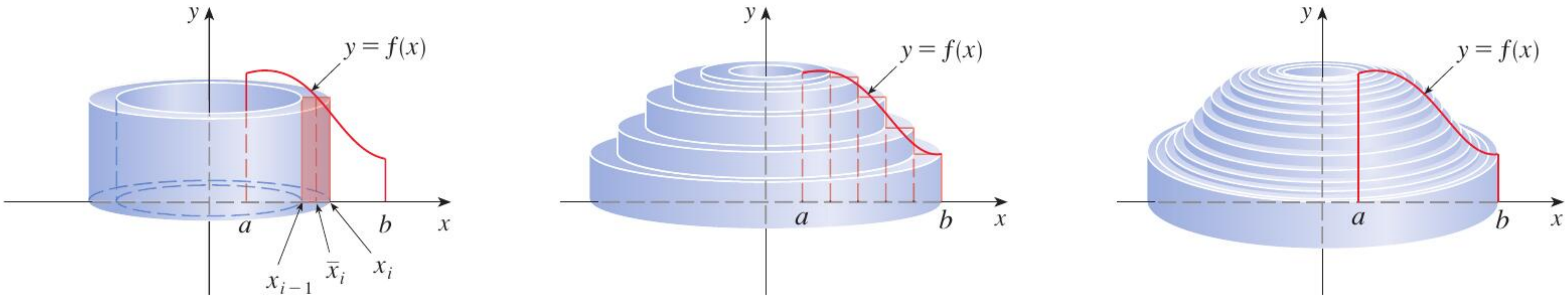
$$\int_a^b 2\pi x \, f(x) \, dx$$



N.B.: SHELL METHOD WORKS WHEN INTEGRATE $\perp$ TO THE AXIS OF ROTATION

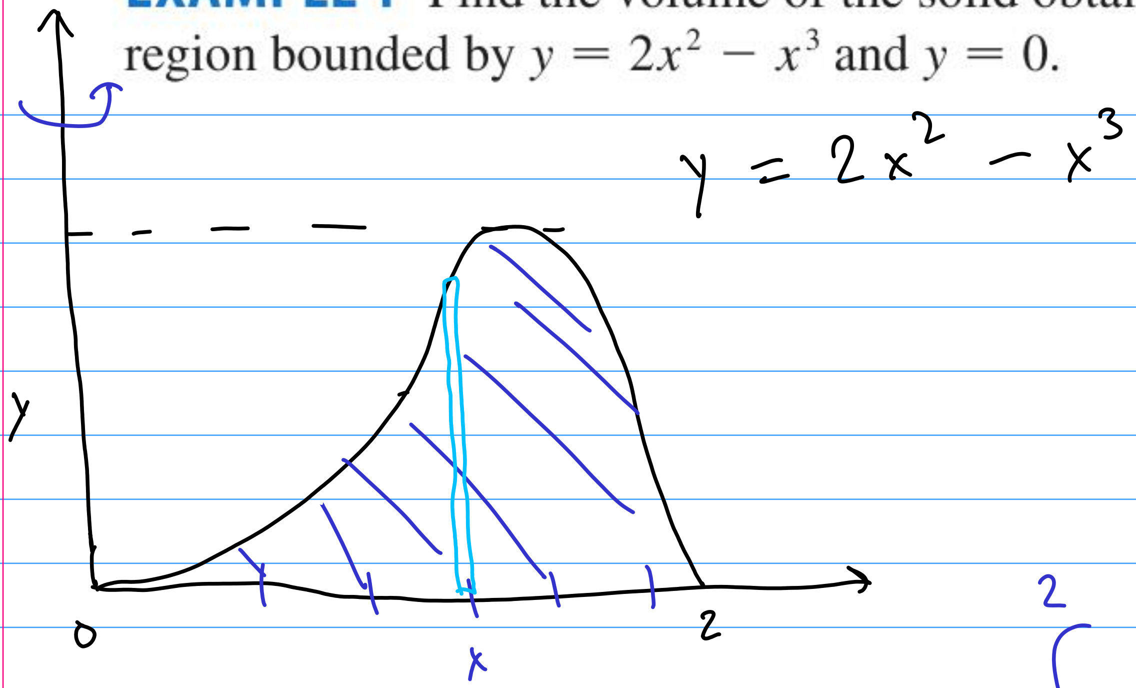


N.B.: WASHER/DISK METHOD WORKS WHEN INTEGRATING $\parallel$ TO THE AXIS OF ROTATION



y
y = f(x)
a
b
x
x_{i-1}
x̄_i
x_i
y
y = f(x)
a
b
x
y
y = f(x)
a
b
x

EXAMPLE 1 Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

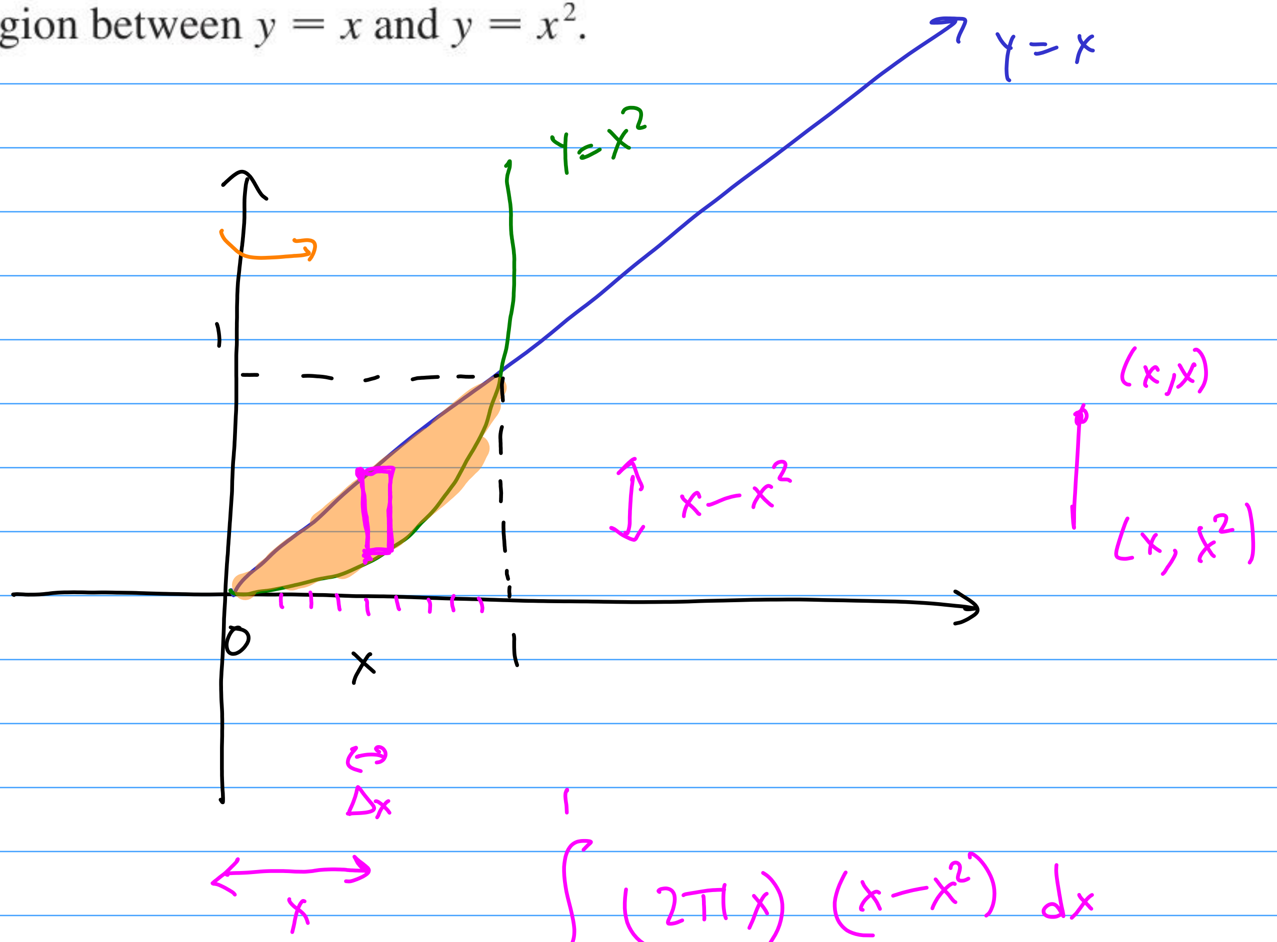


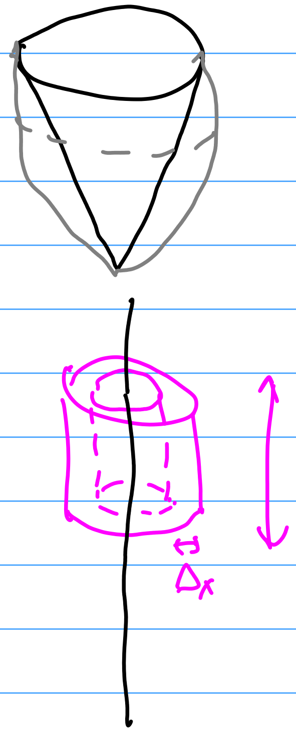
$f(x) = 2x^2 - x^3$

$$\int_0^2 (2\pi(x)) f(x)\,dx$$

$$= \int_0^2 2\pi x \left(2x^2 - x^3\right) dx$$

EXAMPLE 2 Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.





$$\int_0^1 (2\pi x)\,(x - x^2)\,dx$$

$$\int_0^1 (2\pi x)(x-x^2)\,dx = \int_0^1 \left(2\pi x^2 - 2\pi x^3\right)dx$$

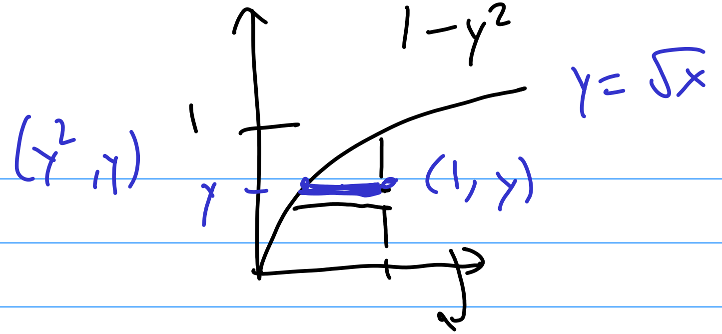
$$= \left[\frac{2\pi x^3}{3} - \frac{2\pi x^4}{4}\right]_0^1$$

$$= \frac{2\pi}{3} - \frac{2\pi}{4} = \frac{\pi}{6}$$

BREAK TILL

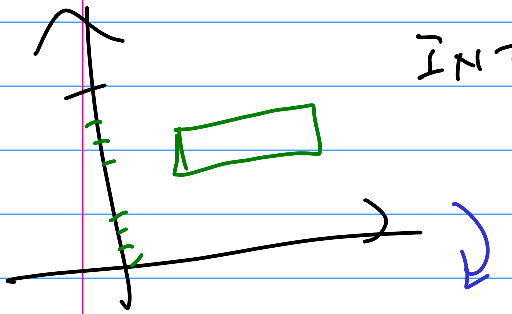
6:41 PM

$$\int_0^1 (2\pi)(y-y^3)\,dy$$

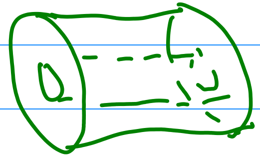


BREAKOUT ROOM

EXAMPLE 3 Use $\boxed{\text{cylindrical shells}}$ to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.



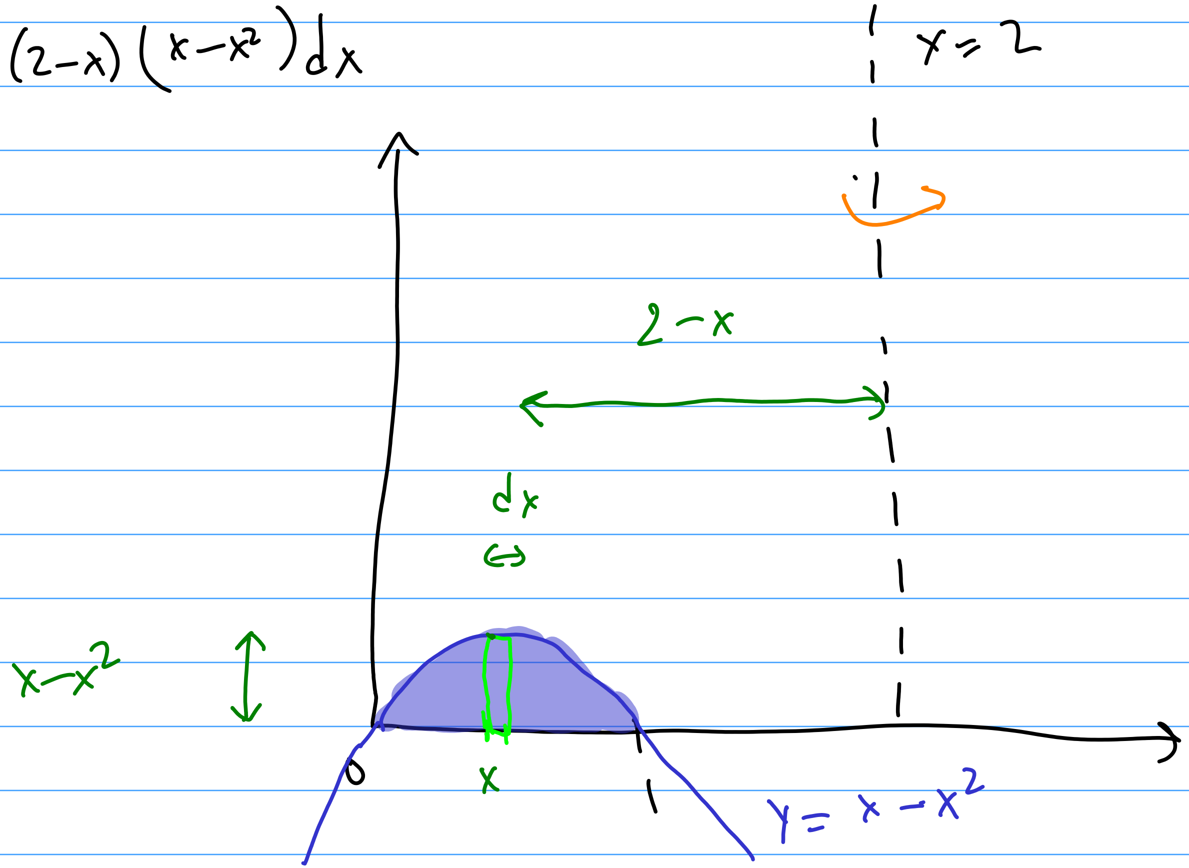
INTEGRATING ALONG THE x-axis

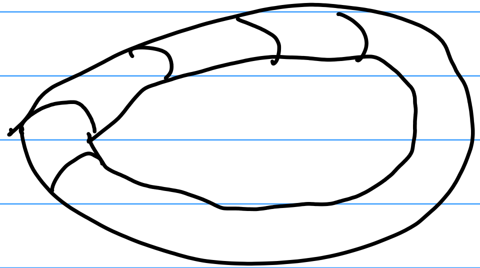


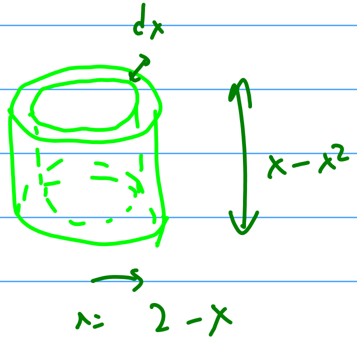
HEIGHT, THICKNESS, CIRCUMFERENCE/RADIUS

EXAMPLE 4 Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $\boxed{x = 2.}$

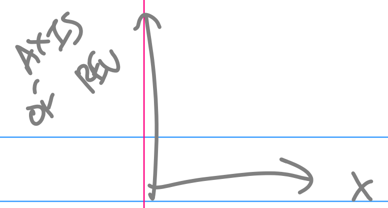
$$\int_0^1 2\pi\,(2-x)\left(x-x^2\right)dx$$







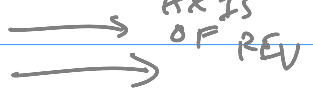
SHELL



EASY

QUESTION: WHICH METHOD TO USE?

WASHER



EASY

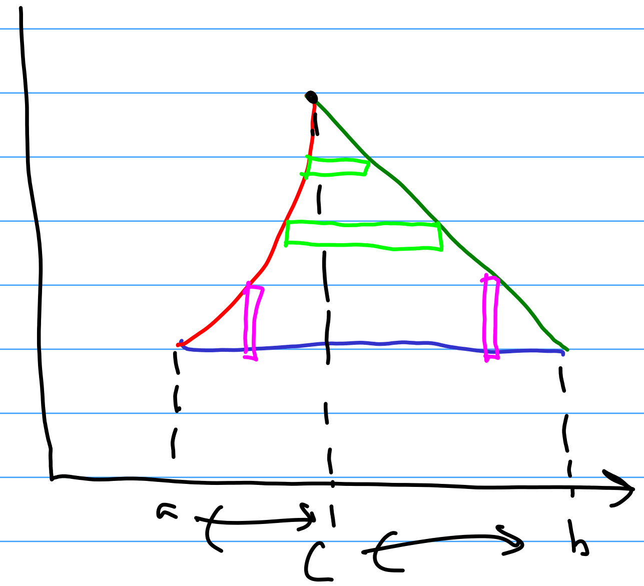
ANSWER: DEPENDS!

KEY FACTOR → IS IT EASIER TO INTEGRATE x OR y?

1. $y = f(x)$ INTEGRATE ALONG x OR $x = g(y)$ INTEGRATE ALONG y ?

2. DOES ONE NEED SEVERAL INTEGRALS?

2. DOES ONE NEED SEVERAL INTEGRALS?



INTEGRATE AGAINST Y

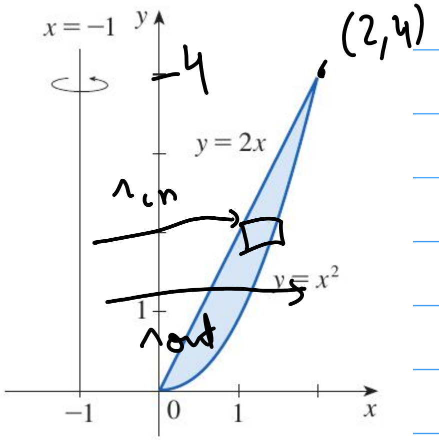
$$\int_a^c \text{RED} + \int_c^b \text{GREEN}$$

SOMETIMES EITHER WORKS.

BREAKOUT ROOM

EXAMPLE 5 Figure 11 shows the region in the first quadrant bounded by the curves $y = x^2$ and $y = 2x$. A solid is formed by rotating the region about the line $x = -1$. Find the volume of the solid using (a) x as the variable of integration and (b) y as the variable of integration.

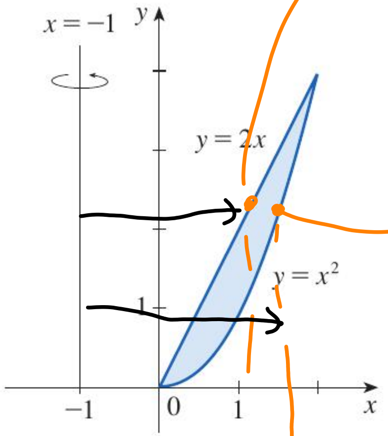
$$\int_0^2 (2\pi)(2x-x^2)(x+1)\,dx$$



y- VARIABLE

WASHER

$$\int_0^4 \pi\left(r_{out}^2 - r_{in}^2\right)\,dy$$



$y=2x \Rightarrow x = y/2$

$y = x^2$

$\Rightarrow x = \sqrt{y}$

$(\sqrt{y}, y)$

$r_{out} = \sqrt{y} + 1$

$(y/2, y)$

$r_{in} = y/2 + 1$

$$\int_0^4 \pi \left[(\sqrt{y}+1)^2 - \left(\frac{y}{2}+1\right)^2 \right] dy$$

$\boxed{\S\ 6.4 \text{ WORK}}$

FORCE

NEWTON'S 2nd LAW OF MOTION:

$$F = \underset{\sim}{m} \tilde{a}$$

$a \rightarrow$ ACCELERATION

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

$m \rightarrow$ MASS

units : N (NEWTON)

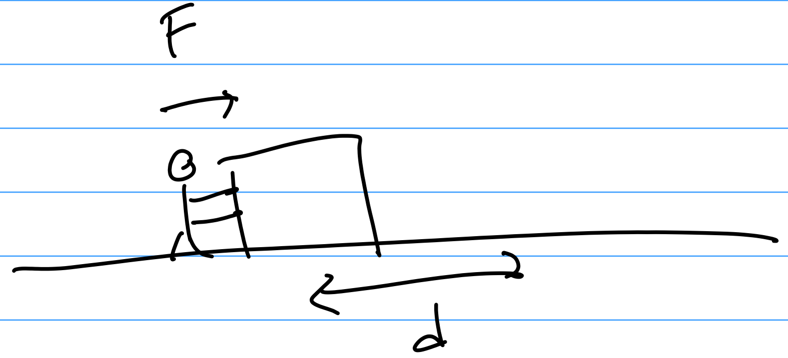
$$N = (kg)(m/s^2) = kgm/s^2$$

DEFN: WORK

(CONSTANT)

→ FORCE = F IN A DIRECTION

→ DISTANCE = d



$$W = F d$$

(WORK)

unit JOULE $J = (N\,m) = kg\,m^2/s^2$

EXAMPLE 1

(a) How much work is done in lifting a 1.2-kg book off the floor to put it on a desk that is 0.7 m high? Use the fact that the acceleration due to gravity is $g = 9.8 \text{ m/s}^2$.

(b) How much work is done in lifting a 20-kg weight 6 m off the ground?

$(100 - 2)(10 + 2)$

$1000 - 20 + 200 - 4$

$1200 - 24 = 1176$

$F_b = (ma)$

$= (20)(9.8)$

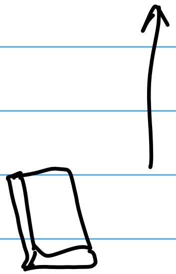
$F_b = 196 \text{ N}$

$d = 6 \text{ m}$

$W_b = (196)(6) \text{ J}$

$= 1200 - 24$

$= 1176$



0.7m

$d = 0.7$

$F = (m)(a)$

$F = (1.2)(9.8)$

$F = 11.76$

$W = Fd = (11.76)(0.7)$