MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY
OFF HRS: M, T, F 4-5PM ;
BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

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COURSE PAGE: bit.ly/sahay142

ANNOUNCEMENTS

1. WEBWORK DEADLINES: (a) WW 7 → TUESDAY, 11 PM
   (b) WW 8 → FRIDAY, 11 PM
   (c) WW 9–12 → SEE SCHEDULE.

2. NEW OFFICE HOURS. (T: 4–5 PM)

3. SAMPLE MIDTERM II WILL BE UPLOADED SOON.

§ 6.4 WORK

RECAP: $F = ma$ [2nd LAW OF NEWTONIAN MOTION]

(units : $N = kg\ m/s^2$)

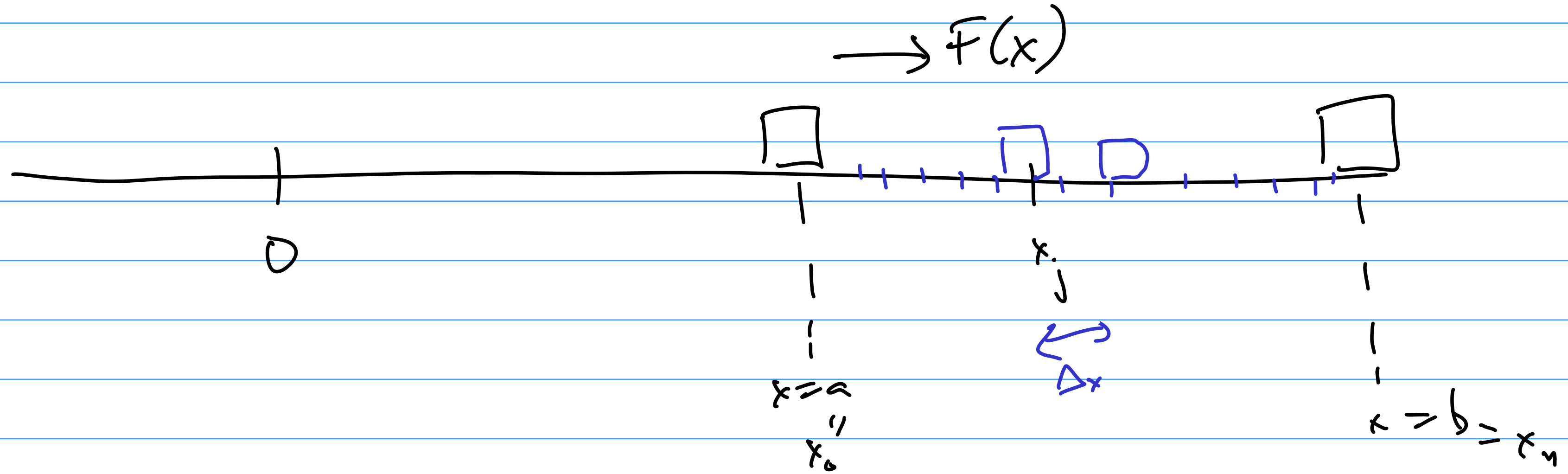
$W = F d$ [DEFN. OF WORK) (units : $J = N m$)

$= kg\ m^2/s^2$

WORK DONE BY AN AGENT MOVING AN OBJECT THROUGH A DISPLACEMENT d WHEN ACTED ON BY A FORCE F.

CONSTANT

WHAT IF F IS NOT CONSTANT?



IN TIME $\Delta t \ll 1$, PARTICLE MOVES Δx

$x \to x + \Delta x$

$$\boxed{\Delta W = F(x)\,\Delta x}$$

$F(x) \approx F(x + \Delta x)$

$$W = \sum_{j=1}^{n} F(x_j)\,\Delta x \qquad \left[\Delta x = \frac{b-a}{n}\right.$$

$x_0 = a$

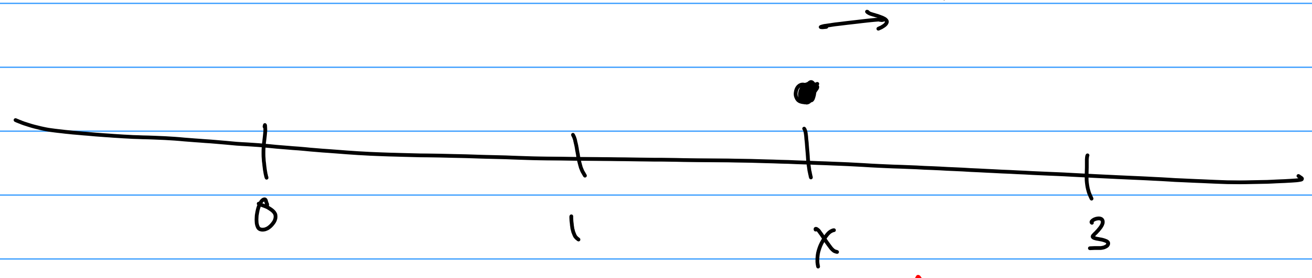
$x_j = b$

$$\xrightarrow[\Delta x \to 0]{n \to \infty} \boxed{\int_a^b F(x)\,dx}$$

W BY A VARIABLE FORCE MOVING A PARTICLE FROM $x = a$ TO $x = b$

EXAMPLE 2 When a particle is located a distance x feet from the origin, a force of $x^2 + 2x$ pounds acts on it. How much work is done in moving it from $x = 1$ to $x = 3$?

$F(x) = x^2 + 2x$



$\leftrightarrow \Delta x$

$$\sum_{j=1}^{n} F(x_j)\,\Delta x \longrightarrow \int_1^3 F(x)\,dx$$

$x = 1$ to $x = 3$

$$W = \int_1^3 (x^2 + 2x)\,dx = \left[\frac{x^3}{3} + x^2\right]_1^3$$

BREAKOUT ROOM 1

4. A variable force of $4\sqrt{x}$ newtons moves a particle along a straight path when it is x meters from the origin. Calculate the work done in moving the particle from $x = 4$ to $x = 16$.

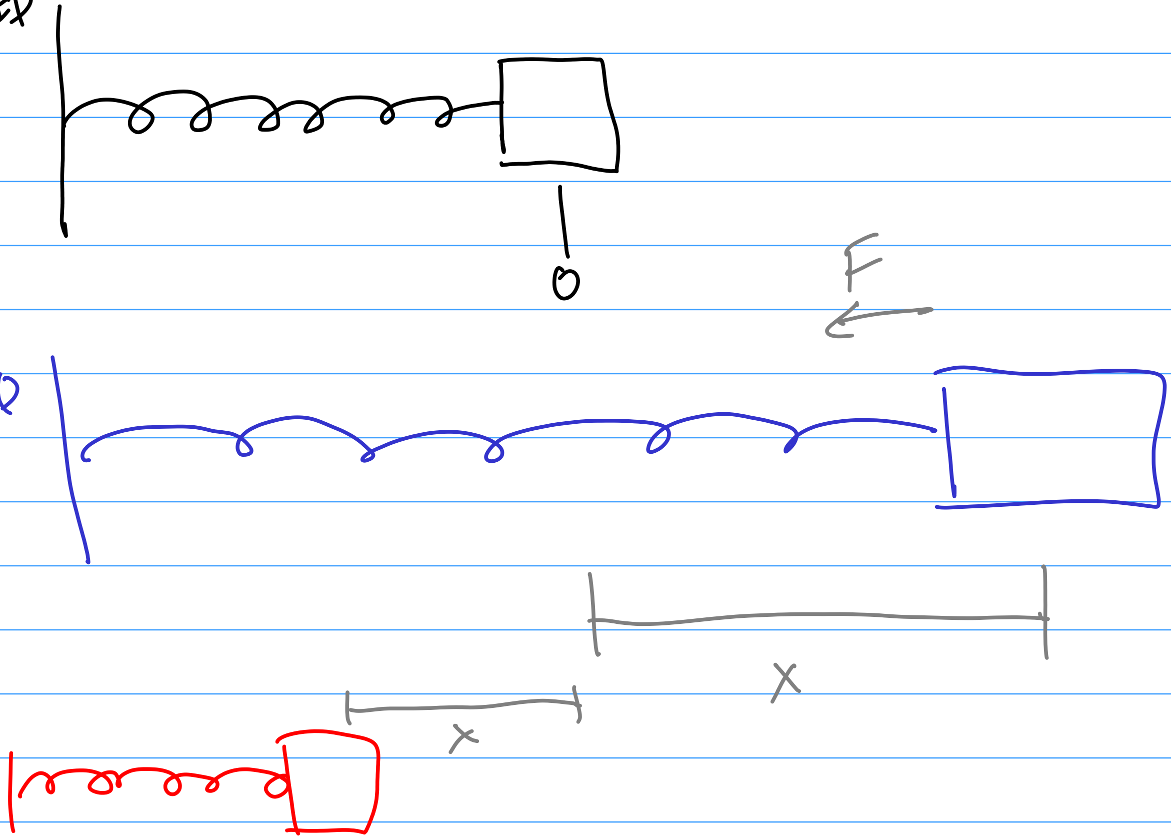
$$\int_4^{16} 4\sqrt{x}\, dx = 4 \left.\frac{x^{3/2}}{3/2}\right]_4^{16}$$

$$= \frac{8}{3}\left[16^{3/2} - 4^{3/2}\right] = \frac{8}{3}\left[4^3 - 2^3\right]$$

HOOKE'S LAW

k (SPRING CONSTANT)

UNSTRETCHED STRING



STRETCHED STRING

$F = kx$

WORK b/w $x = a$ & $x = b$ $= \int_a^b F(x)\, dx$

$= \int_a^b (kx)\, dx$

$= \frac{1}{2} k b^2 - \frac{1}{2} k a^2$

[CONVENTION
$x = 0$ MEANS
THE SPRING
IS IN
THE NATURAL
STATE]

FORCE WHEN STRETCHED x $= kx$ (IN MAGN.)

WORK NEEDED TO STRETCH BY x $= \frac{1}{2} k x^2$

$(a=0, b=x)$

EXAMPLE 3 A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

$k \to ??$ SPRING CONSTANT

$1\,\text{cm} = 0.01\,\text{m}$

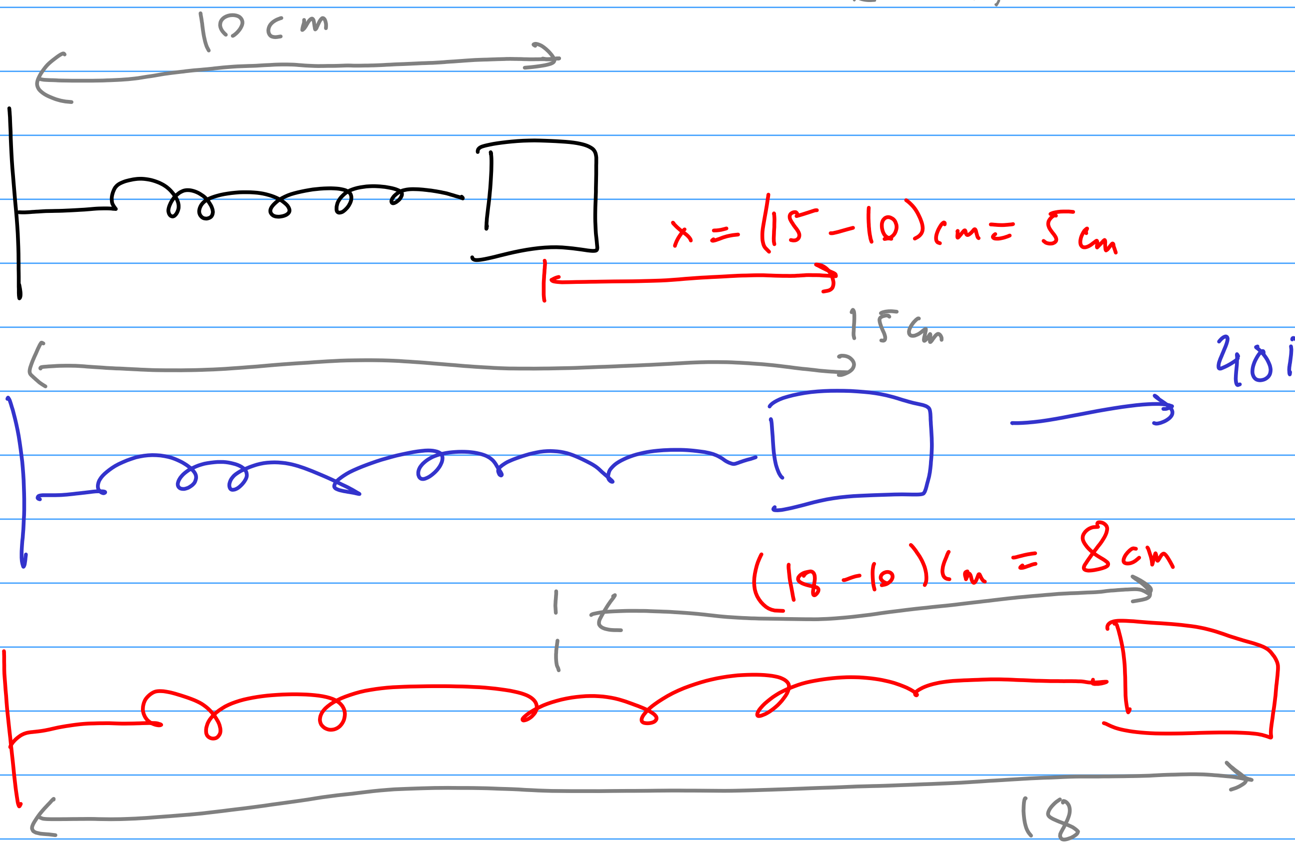
$F = kx$

$(40\text{N}) = (k)(5\,\text{cm})$

$(40\text{N}) = k(0.05\,\text{m})$

$k = \frac{40}{0.05} = \frac{8}{0.01} = 800$

$\boxed{k = 800\ \text{N/m}}$



$a = 5\,\text{cm} = 0.05\,\text{cm}$

$b = 8\,\text{cm} = 0.08\,\text{m}$

w b/w

$x = a$ &

$x = b$

$$\int_a^b F(x)\,dx = \int_a^b (k\,x)\,dx$$

N/m

$$= \int_{0.05}^{0.08} (800)\,x\,dx$$

$$= 800\,\frac{x^2}{2}\Bigg]_{0.05}^{0.08}$$

$$= (400)\left[0.08\right)^2 - (400)\,(0.05)^2$$

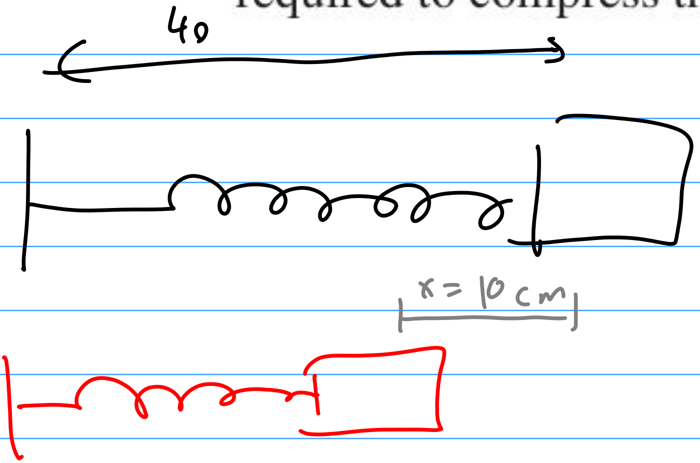
BREAKOUT ROOM 2

$$k = \frac{60}{0.1} = 600 \text{ N/m}$$

8. A spring has a natural length of 40 cm. If a 60-N force is required to keep the spring compressed 10 cm, how much work is done during this compression? How much work is required to compress the spring to a length of 25 cm?

$a = 0$ cm

$b = -10$ cm



$$\int_0^{-0.1} (600x)\,dx$$

$$\frac{600x^2}{2}\Bigg]_0^{-0.1} = (300)(0.1)^2 = 3J$$

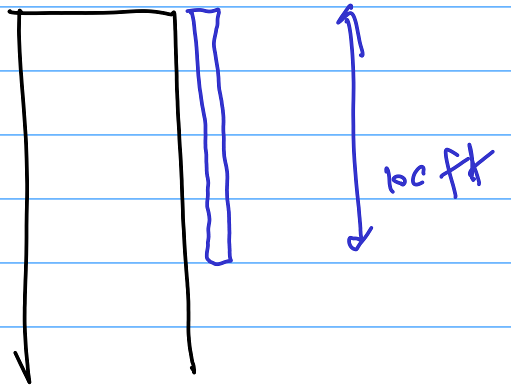
BREAK TILL 6:55 PM EST

$200\,\text{lb}/100\,\text{ft} = 2\ \text{lb/ft}$

EXAMPLE 4 A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building.

(a) How much work is required to lift the cable to the top of the building?

(b) How much work is required to pull up only 20 feet of the cable?



$\square \updownarrow$ 1 ft

$\frac{200}{100} = 2$ lb

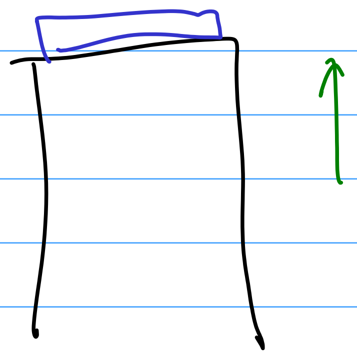
200 lb

↑

UNIT OF FORCE

lb_f (POUND-FORCE)

WEIGHT ON A Δx-LENGTH PIECE $= \frac{200}{100}\Delta x = 2\cdot\Delta x$

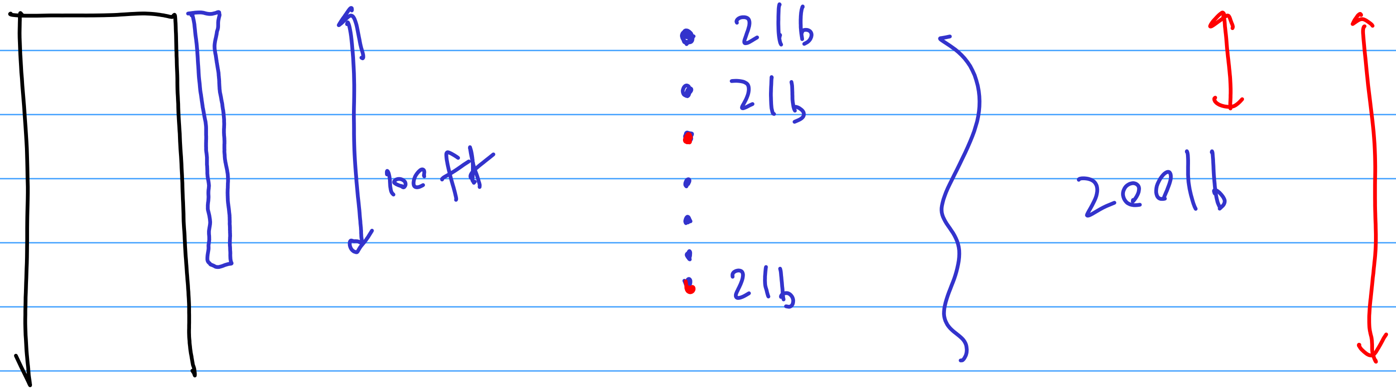


$F = 200$ lb

$\boxed{d} = 100$ ft

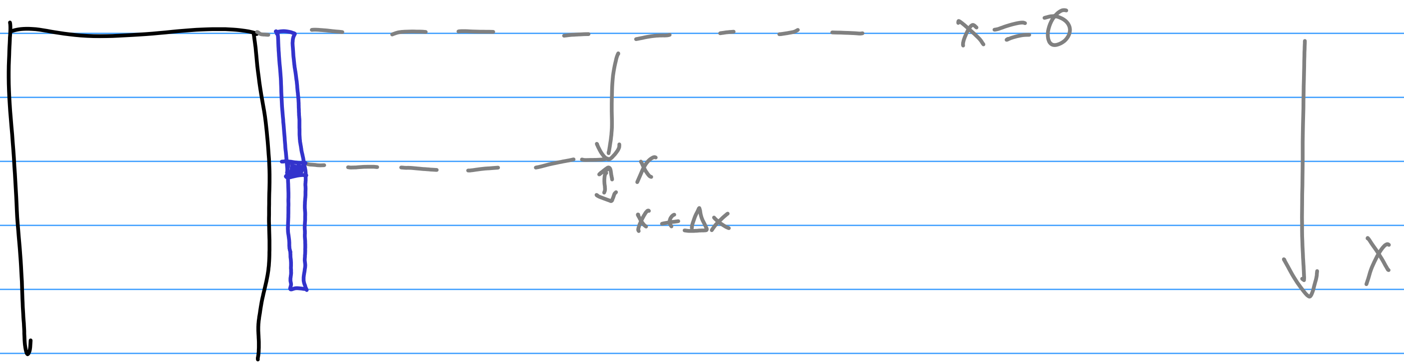
$W = Fd = (200 \times 100)$

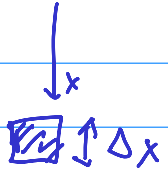
$= 20000$ lb-ft



NEED A

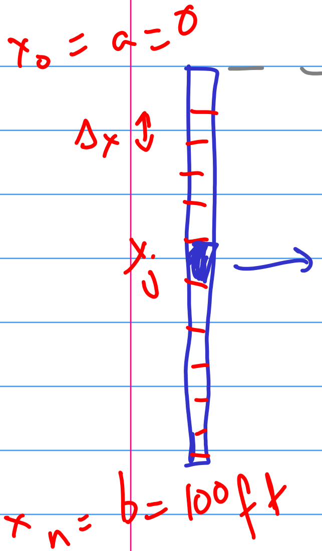
RIEMANN SUM





$F(x) \simeq 2 \cdot \Delta x$

W ON THAT PIECE $= [F(x)](x) \approx (2 \cdot \Delta x) \cdot (x)$

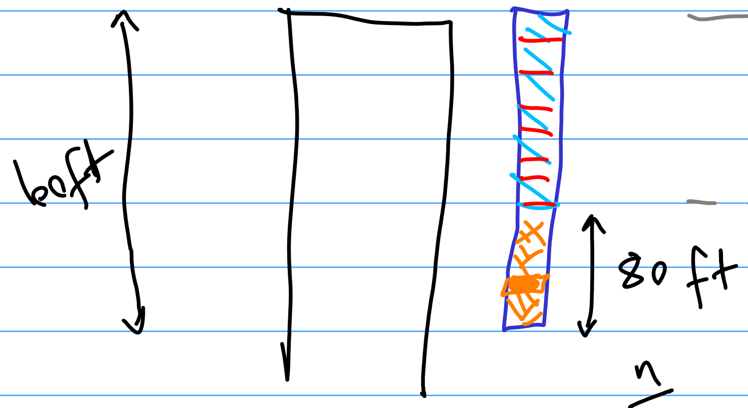


$$2x\,\Delta x$$

$$2x_j\,\Delta x$$

$$\text{TOTAL WORK} \approx \sum_{j=1}^{n} 2x_j\,\Delta x$$

$$\xrightarrow[\Delta x\to 0]{n\to\infty} \int_0^{100} 2x\,dx = x^2\Big]_0^{100} = 10000 \text{ ft-lb}$$

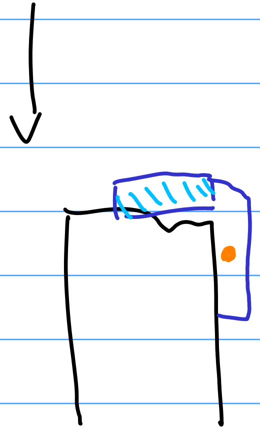


$x = 0$

$x = 20$

$$\sum_{j=1}^{n} (x_j)(2\Delta x) \longrightarrow \int_0^{20} 2x\,dx$$

WORK FOR THE LIGHT BLUE PART



$$F = (200)\left(\frac{80}{100}\right)$$

$$20 \uparrow d \Rightarrow$$

WORK FOR THE ORANGE PART

$$(200)\left(\frac{80}{100}\right)(20)$$

BREAKOUT ROOM 3

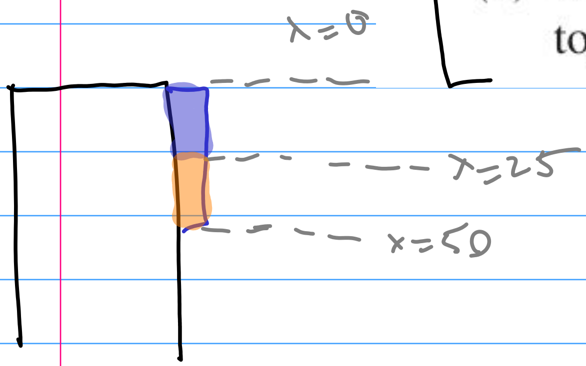
$50 \times 0.5 = 25$ lb

$(\Delta x)(0.5)$

13. A heavy rope, 50 ft long, weighs $\boxed{0.5\text{ lb/ft}}$ and hangs over the edge of a building ~~120 ft high~~.

~~(a)~~ How much work is done in pulling the rope to the top of the building?

(b) How much work is done in pulling half the rope to the top of the building?



$\underbrace{(0.5)(\Delta x)}_{\text{FORCE}}$ $\quad x$ DISTANCE

$$\int_0^{25} 0.5\, x\, dx$$

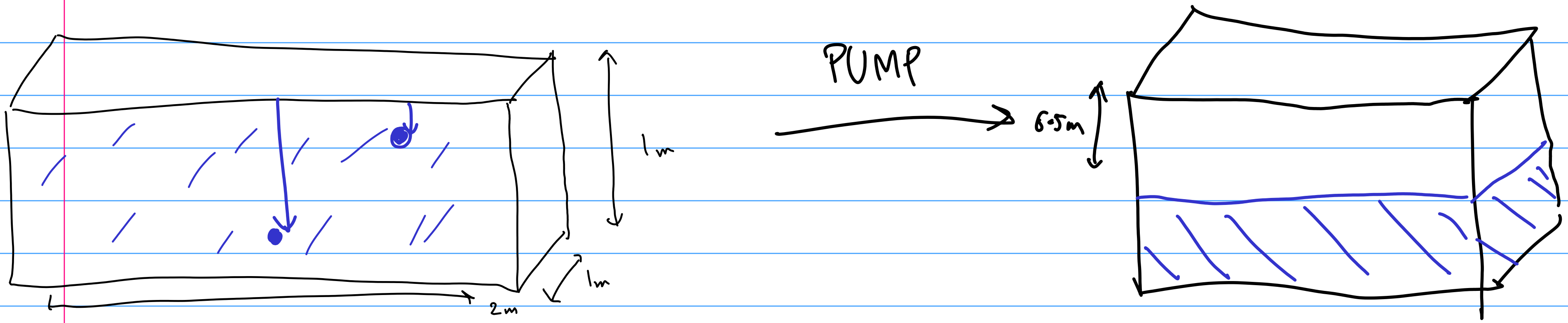
FORCE $(0.5)(25)$

DISTANCE 25

$\to (0.5)(25)(25)$

≈ 312.5

21. An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that the density of water is $1000\ \mathrm{kg/m^3}$.)



$V = (2m)(1m)(1m) = 2\ m^3 \Rightarrow$ VOLUME REMOVED $= 1\ m^3 \Rightarrow$ MASS REMOVED $=$ DENSITY $\times$ VOLUME $= 1000$ kg

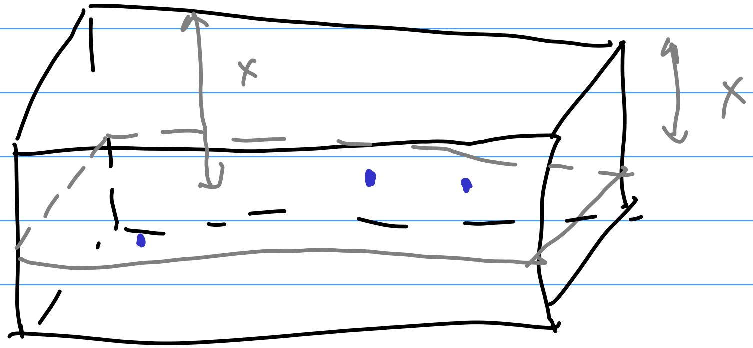
$m = 1000 \text{ kg}$ $g = 9.8 \text{ m/s}^2$

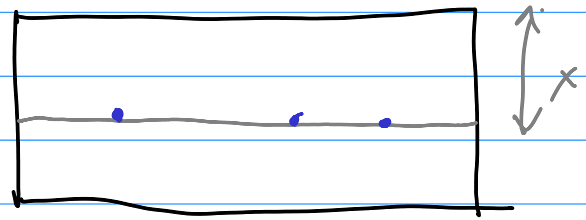
TOTAL $F = mg = (1000 \times 9.8) = 9800 \text{ N}$

$d = 0.5 \text{ m}$

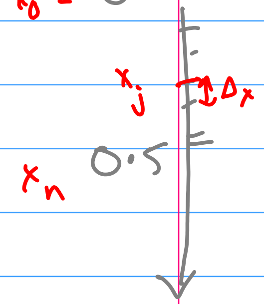
$W = Fd = (9800)(0.5) = 4900 \text{ J}$

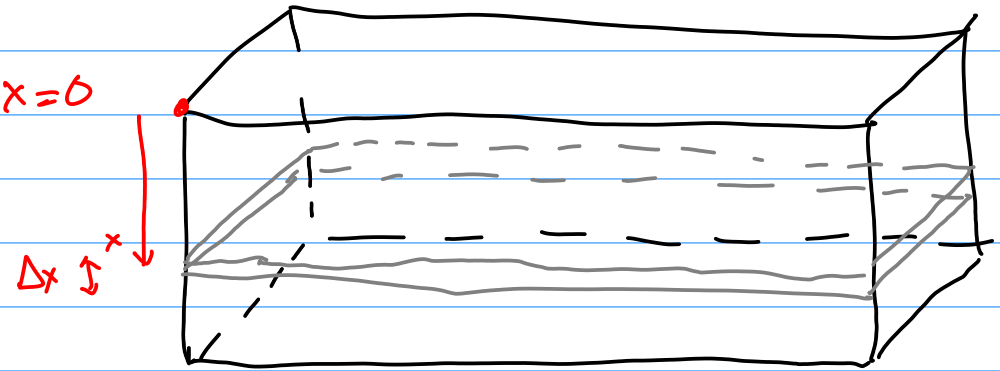
$\times$

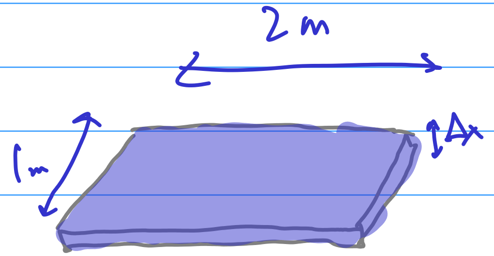

x
x


x

$x \rightarrow$ DEPTH FROM THE TOP







VOLUME $= 2\Delta x$

MASS $= (2\Delta x)(\text{DENSITY})$

$= (2\Delta x)(1000)$

$= 2000\,\Delta x$

FORCE $= mg = (2000\Delta x)(9.8)$

$= 19600\,\Delta x$

WORK FOR A PIECE AT DEPTH x $= F \times d$

$= (19600\,\Delta x)(x)$

$= 19600\,x\,\Delta x$

$$\text{WORK DONE} \simeq \sum_{j=1}^{n} 19600\, x_j \,\Delta x$$

$$\xrightarrow[\Delta x \to 0]{n \to \infty} \int_0^{0.5} 19600\, x \, dx$$

$$= 19600 \left. \frac{x^2}{2} \right]_0^{0.5}$$

$$= (9800)(0.5)^2 = 2450 \text{ J}$$