MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

OFF HRS: M, T, F 4-5PM ; BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

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COURSE PAGE : bit.ly/sahay142

ANNOUNCEMENTS

1. WEBWORK DEADLINES: (a) WW 7 → TODAY, 11 PM
   (b) WW 8 → FRIDAY, 11 PM
   (c) WW 9–12 → SEE SCHEDULE.

2. SAMPLE EXAM – TONIGHT

§ 7.1 INTEGRATION BY PARTS

(AKA REVERSE PRODUCT RULE)

CONTD.

RECALL

$$\int u\,dv = uv - \int v\,du$$

DIFFERENTIATE

$u = ?$ $\Longrightarrow$ du

$dv = ?$ $\Longrightarrow$ v

INTEGRATE

EXAMPLE 4 Evaluate $\int e^x \sin x \, dx$.

$u = \sin x \Rightarrow du = \cos x \, dx$

$dv = e^x dx \Rightarrow v = e^x$

$$\int \underbrace{e^x}_{dv?} \underbrace{\sin x}_{u} \, dx = e^x \sin x - \int \underbrace{e^x}_{v} \underbrace{\cos x \, dx}_{du}$$

dv

u

$$(*) \quad \int e^x \cos x \, dx = e^x \cos x - \int e^x (-\sin x) \, dx$$

$$I = \int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

$$= e^x \sin x - \left[e^x \cos x - \int (-e^x \sin x) \, dx \right]$$

$$= e^x \sin x - e^x \cos x - \underbrace{\int e^x \sin x \, dx}_{I}$$

$e^{ax} \sin bx$

OR

$e^{ax} \cos bx$

$$I = e^x \sin x - e^x \cos x - I \Rightarrow 2I = e^x \sin x - e^x \cos x$$

BLACK MAGIC

$$\Rightarrow I = \frac{1}{2}\left[e^x \sin x - e^x \cos x \right] + C$$

BREAKOUT ROOMS

$u = y$

$dv = e^{-y}\,dy$

$v = -e^{-y}$

$y(-e^{-y}) - e^{-y} + C$

6. $\int ye^{-y}\,dy$

$dv = w\,dw \Rightarrow v = \frac{w^2}{2}$

$u = \ln w$

9. $\int w \ln w\,dw$

$\left(\frac{w^2}{2}\right)\ln w - \frac{1}{4}w^2 + C$

11. $\int (x^2 + 2x)\cos x\,dx$

$u = x^2 + 2x$

$dv = \cos x\,dx$

22. $\int e^x \sin \pi x\,dx$

$u = \sin \pi x$	$u = e^x$
$dv = e^x\,dx$	$dv = \sin \pi x\,dx$

$$(x^2+2x)\sin x - \int (2x+2)\sin x\, dx$$

$u = 2x+2, \quad dv = \sin x\, dx$

$du = 2dx, \quad v = -\cos x$

$$\int (2x+2)\sin x\, dx = \boxed{\int 2x \sin x\, dx} + \underbrace{\int 2\sin x\, dx}_{-2\cos x}$$

$$\int e^x \sin \pi x \, dx = \frac{-e^x \cos \pi x}{\pi} - \int e^x \left(-\frac{\cos \pi x}{\pi} \right) dx$$

$$u = e^x \quad \Rightarrow \quad du = e^x \, dx$$

$$dv = \sin \pi \, dx \quad \Rightarrow \quad v = \frac{-\cos \pi x}{\pi}$$

$$v = \int dv = \int \sin \pi x \, dx = \int \frac{\sin u \, du}{\pi} = \frac{-\cos u}{\pi} = \frac{-\cos \pi}{\pi}$$

$$u = \pi x$$

$$du = \pi \, dx$$

$$\frac{d}{dx} (\cos \pi x) = \pi (-\sin \pi x)$$

$$I = \int e^x \sin \pi x \, dx = \frac{-e^x \cos \pi x}{\pi} - \int e^x \left(-\frac{\cos \pi x}{\pi} \right) dx$$

$$= \frac{-e^x \cos \pi x}{\pi} + \int e^x \frac{\cos \pi x}{\pi} \, dx$$

$$u = e^x \Rightarrow du = e^x$$

$$dv = \frac{\cos \pi x}{\pi} \, dx \Rightarrow v = \frac{\sin \pi x}{\pi^2}$$

$$= -\frac{e^x \cos \pi x}{\pi} + \frac{e^x \sin \pi x}{\pi^2} - \underbrace{\int e^x \frac{\sin \pi x}{\pi^2} \, dx}_{\frac{1}{\pi^2}(I)}$$

$$I = e^x \left[\frac{\sin \pi x}{\pi^2} - \frac{\cos \pi x}{\pi} \right] - \frac{1}{\pi^2} I$$

$$I\left(1+\frac{1}{\pi^2}\right) = e^x\left[\frac{\sin \pi x}{\pi^2} - \frac{\cos \pi x}{\pi}\right]$$

$$\Rightarrow \quad I = \frac{e^x\left[\frac{\sin \pi}{\pi^2} - \frac{\cos \pi x}{\pi}\right]}{1 + \frac{1}{\pi^2}}$$

$$\int e^x \sin \pi x \, dx = e^x(\sin \pi x) - \int \underbrace{e^x(\pi \cos \pi x)\, dx}$$

$u = \sin \pi x \Rightarrow du = \pi \cos \pi x$

$dv = e^x \, dx \Rightarrow v = e^x$

$u = \pi \cos \pi x$

$dv = e^x dx$

$du = -\pi^2 \sin \pi x \, , \; v = e^x$

$$\int e^x \pi \cos \pi x \, dx = e^x(\pi \cos \pi x) - \int e^x(-\pi^2 \sin \pi x)\, dx$$

$$I = \int e^x \sin \pi x = e^x \sin \pi x - \pi e^x \cos \pi x - \underbrace{\int \pi^2 (e^x \sin \pi x)\, dx}_{\pi^2 (I)}$$

$$I = e^x \sin \pi x - \pi e^x \cos \pi x - \pi^2 I$$

$$I(1+\pi^2) = e^x [\sin \pi x - \pi \cos \pi x \quad)$$

$$I = e^x \left[\frac{\sin \pi x - \pi \cos \pi x}{1+\pi^2} \right]$$

BREAK TILL 7:05 PM

(DEFINITE) INTEGRATION BY PARTS

(INDEFINITE)

$$\int_a^b f(x)\,g'(x)\,dx = f(x)\,g(x)\Big]_a^b - \int_a^b f'(x)\,g(x)\,dx$$

(Pf: INDEFINITE INTEGRATION BY PARTS

+ FUNDAMENTAL THEOREM.

EXAMPLE 5 Calculate $\int_0^1 \tan^{-1} x\, dx$.

$$\int_0^1 \tan^{-1} x \, dx$$

$$= \underbrace{x \tan^{-1} x \Big]_0^1}_{uv} - \overbrace{\int_0^1 \frac{x}{1+x^2}\, dx}^{\int v\, du}$$

$u = \tan^{-1} x$

$dv = dx$

$du = \left(\frac{dx}{1+x^2}\right)$

$v = x$

$$= \tan^{-1} 1 - \int_0^1 \left(\frac{x}{1+x^2}\right) dx$$

$u = 1 + x^2$

$du = 2x\, dx$

$$\tan^{-1} 1 - \int_0^1 \left(\frac{x}{1+x^2} \right) dx$$

u, $\frac{du}{2}$

$u = 1 + x^2$

$x = 0, \; u = 1$

$x = 1, \; u = 2$

$$= \tan^{-1} 1 - \int_1^2 \frac{du/2}{u}$$

$$= \tan^{-1} 1 - \frac{1}{2} \int_1^2 \frac{du}{u}$$

$$= \tan^{-1} 1 - \frac{1}{2} \ln u \Big]_1^2 = \tan^{-1} 1 - \frac{\ln 2}{2}$$

$?\; // \; \frac{\pi}{4} - \frac{\ln 2}{2}$

BREAKOUT ROOMS

33. $\int_1^5 \frac{\ln R}{R^2}\, dR$

34. $\int_0^{2\pi} t^2 \sin 2t\, dt$

2. u, dv

1. $u = \ln R \Rightarrow du = 1/R$

$dv = \frac{dR}{R^2} \Rightarrow v = \int \frac{dR}{R^2} = -\frac{1}{R}$

$= \int R^{-2}\, dR = \frac{R^{-1}}{-1} = \frac{-1}{R}$

$u = t^2 \Rightarrow du = 2t\,dt$

$dv = \sin 2t\,dt$,

$v = \int \sin 2t\,dt$

$= \frac{-\cos 2t}{2}$

$$\int_1^5 \frac{\ln R}{R^2}\, dR = -\frac{\ln R}{R}\Big]_1^5 - \int_1^5 \left(-\frac{1}{R^2}\right) dR$$

$$= \frac{-\ln R}{R} - \frac{1}{R}\Big]_1^5$$

$$= \frac{-\ln 5}{5} - \frac{1}{5} + 1$$

$$= \frac{4 - \ln 5}{5}$$

34. $\int_0^{2\pi} t^2 \sin 2t \, dt$

$$\int_0^{2\pi} t^2 \sin 2t \, dt = t^2\left(-\frac{\cos 2t}{2}\right)\Bigg]_0^{2\pi} - \int_0^{2\pi} (2t)\left(-\frac{\cos 2t}{2}\right) dt$$

$$u = t^2 \quad \Rightarrow \quad du = 2t\,dt$$

$$dv = \sin 2t \, dt \quad \Rightarrow \quad v = -\frac{\cos 2t}{2}$$

$$= (2\pi)^2\left(\frac{-\cos 2\pi}{2}\right) + \int_0^{2\pi} t \cos 2t \, dt$$

$$= -2\pi^2 + \int_0^{2\pi} t \cos 2t \, dt$$

$u = t \Rightarrow du = dt$

$dv = (\cos 2t)\, dt \Rightarrow v = \int \cos 2t \, dt$

$$= \frac{\sin 2t}{2}$$

$\int f(ax)\, dx = \frac{F(ax)}{a}$ $[F(x) = \int f(x)\, dx$

$u = ax$

$$= -2\pi^2 + \underbrace{\left[\frac{t \sin 2t}{2}\right]_0^{2\pi}}_{(=0)} - \int_0^{2\pi} \frac{\sin 2t}{2}\, dt$$

$$= -2\pi^2 - \int_0^{2\pi} \frac{\sin 2t}{2}\, dt$$

$$-2\pi^2 - \int_0^{2\pi} \frac{\sin 2t}{2}\, dt \qquad [\because u = 2t]$$

$$= -2\pi^2 - \frac{1}{2}\left(-\frac{\cos 2t}{2}\right)\Big]_0^{2\pi}$$

$$= -2\pi^2 - \frac{1}{4}\Big[\underbrace{-\cos 4\pi + \cos 0}_{=0}\Big]$$

$$= -2\pi^2$$