

Math 142: Calculus II

Sample Midterm 2

June 17th, 2021

NAME (please print legibly): _____

Your University ID Number: _____

- The exam will be 75 minutes long. You will get extra time in the end to upload the exam to Gradescope.
- There are 9 pages.
- A formula sheet is provided.
- No calculators, phones, electronic devices, books, notes are allowed during the exam. The only materials you are allowed to use are pen/pencil and paper. In particular, you are NOT allowed to take the exam on a tablet.
- You are allowed to use a phone or tablet to take photographs of your answer sheet once the exam is over. If you finish early, you must take permission before taking photographs. Once you start taking photographs, you are not allowed to write.
- **Show all work and justify all answers as much as possible.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Numerical or algebraic simplifications of answers are not required.
- In several questions, you do NOT need to evaluate the integral. Please read the questions carefully to ensure you do not waste time trying to compute an integral you do not need to.

QUESTION	VALUE	SCORE
1	0	
2	10	
3	30	
4	20	
5	30	
6	30	
TOTAL	120	

Formulas

1. (0 points) Copy the following honesty pledge on to your answer sheet. Remember to sign and date it.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

2. (10 points)

A particle travels with velocity $f(t) = (t - 6)$ in m/s while travelling along a line for 10 seconds, starting at $t = 0$. Find the time at which the instantaneous velocity of the particle is equal to its average velocity over its motion.

$$f_{\text{avg}} = \frac{1}{10 - 0} \int_0^{10} (t - 6) dt$$

$$\left[\frac{1}{b-a} \int_a^b f(x) dx \right] = \frac{1}{10} \int_0^{10} (t - 6) dt$$

$$f_{\text{avg}} = -1 = \frac{1}{10} \left(\frac{t^2}{2} - 6t \right) \Big|_0^{10}$$

$$f(c) = c - 6 = f_{\text{avg}} = \frac{1}{10} \left(\frac{10^2}{2} - 6 \times 10 \right) = -1$$

$$\Rightarrow c - 6 = -1 \Rightarrow c = 5$$

INTEGRAL MVT APPLIES TO $f(t)$ ON $[0, 10]$

3. (30 points) Consider the region bounded in the first quadrant by $y = x^2$ and $y = x$.

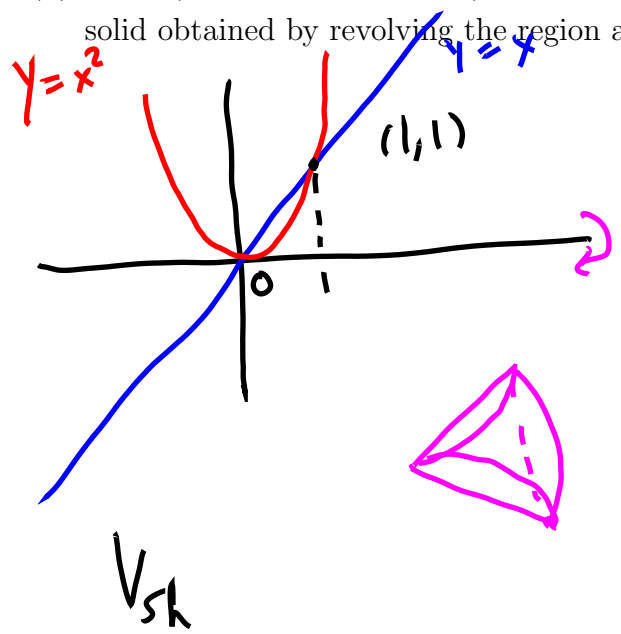
(a) Sketch the curves and shade the region described above.

(b) Write (but do NOT evaluate) an integral that is equal to the area of the region.

(c) Write (but do NOT evaluate) an integral using the ~~shell~~ **WASHER** method for the volume of the solid obtained by revolving the region about the x -axis.

(d) Write (but do NOT evaluate) an integral using the ~~washer~~ **SHELL** method for the volume of the solid obtained by revolving the region about the line $y = 1$.

DONE
IN
NOTES



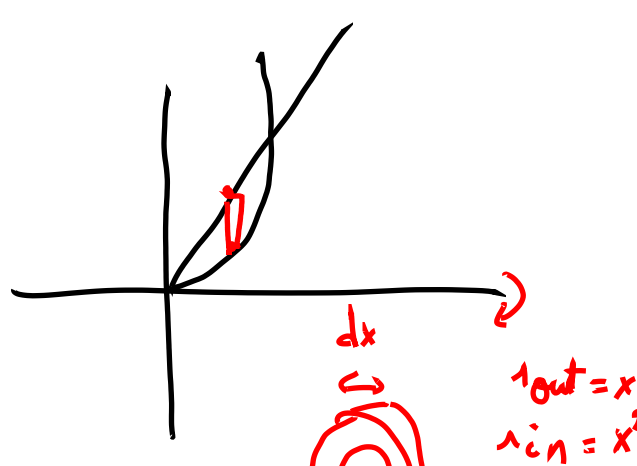
WASHER

$y^2 = x, x = 0 \rightarrow (0,0)$
 $x = 1 \rightarrow (1,1)$

SHELL

$f(x) - g(x)$

$$A = \int_0^1 (x - x^2) dx$$



$$\int_0^1 \pi (x^2 - (x^2)^2) dx = \pi r_{out}^2 h - \pi r_{in}^2 h$$

$$= \int_0^1 \pi (x^2 - x^4) dx$$

4. (20 points)

A spherical tank of radius $r = 2$ m is full of water. The water is pumped out of a hole at the top of the tank over time.

- (a) Write (but do NOT evaluate) an integral that represents the work done to empty the tank.
- (b) Write (but do NOT evaluate) an integral that represents the work done to empty half the tank.

Recall that the density of water is 1000 kg m^{-3} and that the acceleration due to gravity is 9.8 m s^{-2} .

$\pi r^2 dx$

$x = 0$

$x = 2$ (RADIUS = 2)

$x = 4$ (2R)

FORCE ON THIS PIECE

$F = ma$ (2nd LAW)

$= mg = 9800\pi \dots$ $g = 9.8 \text{ m/s}^2$

MASS OF THE PIECE

$m = \rho V = 1000\pi (4 - (2-x)^2) dx$ $\rho = 1000 \text{ kg/m}^3$

VOLUME OF THE PIECE

$= \pi r^2 dx = \pi (4 - (x-2)^2) dx$

$\lambda(x)^2 = 4 - (x-2)^2$

$|2-x|$

$\lambda(x)^2 + (x-2)^2 = 2^2$

5. (30 points)

Compute the following indefinite integrals:

(a)

$$\int \frac{(1+2x)}{(x+x^2)^2} dx$$

$$\int e^u \sin u du$$

(b)

(c)

$$\int (\ln x)^2 dx$$

$$\int u^2 e^u du$$

$$\int (\ln x)^2 dx$$

$$u = (\ln x)^2 \Rightarrow \frac{du}{dx} = \frac{d}{dx} (\ln x)^2 = \frac{2 \ln x}{x}$$

$$dv = dx \Rightarrow v = x \Rightarrow du = \frac{2 \ln x dx}{x}$$

$$= x (\ln x)^2 - \int \left(\frac{2 \ln x}{x} \right) x dx$$

$$= x (\ln x)^2 - \int 2 \ln x dx \quad \int 2 dx$$

$$= x (\ln x)^2 - \left[2x \ln x - \int (2x) \left(\frac{1}{x} \right) dx \right]$$

$$= \boxed{x (\ln x)^2 - 2x \ln x + 2x + C}$$

$$x + x^2 \xrightarrow{\frac{d}{dx}} 1 + 2x$$

$$u = x + x^2 \Rightarrow du = (1 + 2x) dx$$

$$\frac{du}{u^2} = -\frac{1}{u} = -\frac{1}{x+x^2} + C$$

(POWER RULE)

$$u = \ln x \Rightarrow x = e^u$$

$$du = \frac{dx}{x} \Rightarrow \frac{dx}{e^u}$$

$$\rightarrow dx = e^u du$$

6. (30 points)

Compute the following definite integrals:

$$u = \ln v \Rightarrow du = \frac{dv}{v}$$

(a)

$$u = x \Rightarrow du = dx$$

$$dv = \sin x dx \Rightarrow v = (-\cos x)$$

$$\int_1^2 \frac{dv}{v(\ln v)^3} = \int \frac{du}{u^3} = -\frac{1}{2u^2}$$

(b)

$$x(-\cos x) \Big|_0^{2\pi} - \int_0^{2\pi} (-\cos x) dx$$

$$\int_0^{2\pi} x \sin x dx = -2\pi$$

$$\int \frac{du}{u^2} = -\frac{1}{u} \Big|_{v=1}^{v=2} = -\frac{1}{2(\ln v)} \Big|_{v=1}^{v=2}$$

(c)

$$(2\pi)(-1) + \int_0^{2\pi} \cos x dx$$

$$\sin x \Big|_0^{2\pi} = (0 - 0) = 0$$

$$\int_0^1 \arcsin x dx$$

$$u = \arcsin x$$

$$dv = dx \Rightarrow v = x$$

$$u = \ln v$$

$$v = 2 \Rightarrow u = \ln 2$$

$$v = 1 \Rightarrow u = 0$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow du = \frac{dx}{\sqrt{1-x^2}}$$

$$x \arcsin x \Big|_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

$$1. \arcsin(1) = \pi/2$$

$$- \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

$\xrightarrow{\frac{du}{-2}}$

$\xrightarrow{\sqrt{u}}$

$$u = 1 - x^2$$

$$du = -2x dx$$

$$= \pi/2 + \int_1^0 \frac{du}{2\sqrt{u}}$$

$$\begin{matrix} x=0 \\ u=1 \\ x=1 \\ u=0 \end{matrix}$$

$$= \pi/2 + \left[\sqrt{u} \right]_1^0 = \pi/2 - 1$$

$$\int_1^0 \frac{u^{-1/2}}{2} du = \frac{u^{1/2}}{1/2} = \frac{2\sqrt{u}}{2}$$