# MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

OFF HRS: M, T, F 4-5PM ;
BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

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COURSE PAGE: bit.ly/sahay142

CLASS STARTS

AT 7:20 PM EASTERN

# ANNOUNCEMENTS

1. WEBWORK 9 → DUE WEDNESDAY AT 11 PM
   " 10 → " FRIDAY " "
2. PLEASE FILL OUT WEEK 4 FEEDBACK!
3. MIDTERM 2 + OTHER SCORES TO BE UPDATED SOON.
4. NO MORE SUGGESTED PROBLEMS

$$\boxed{\S\, 7.2 \quad \text{TRIGNOMETRIC INTEGRALS}}$$

RECALL: $\boxed{\sin^2\theta + \cos^2\theta = 1}$

GOAL: LEVERAGE THE ABOVE TO SOLVE

$$\int \sin^n\theta \cos^m\theta \, d\theta$$

$n = 0, \ m = 3$

$\int \sin^n x \cos^m x \, dx$

**EXAMPLE 1** Evaluate $\int \cos^3 x \, dx$.

$$\underbrace{\int \cos^3 x \, dx}$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\int (\cos x)(\cos^2 x) \, dx = \int \underbrace{(\cos x)}_{\underbrace{\cos x \, dx}_{du}} \underbrace{(1 - \sin^2 x)}_{1 - u^2} \, dx$$

$u = \sin x$

$\Rightarrow du = \cos x \, dx$

$$= \int (1 - u^2) \, du$$

$$= u - \frac{u^3}{3} + C$$

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3} + C$$

IDEA: $u = \sin x$

ODD POWER OF $\cos x$ $\longrightarrow$ KEEP ONE POWER

$\longrightarrow$ $\cos x \, dx = d(\sin x) = du$

REWRITE THE REST IN TERMS OF $u = \sin x$

$\int \sin^n x \cos^m x \, dx \longrightarrow m$ IS ODD.

**EXAMPLE 2** Find $\int \sin^5 x \cos^2 x \, dx$.

ODD

EVEN

$$\int (\sin^4 x)(\cos^2 x) \sin x \, dx$$

$$\sin^2 x = 1 - \cos^2 x$$

$$= \int (\sin^2 x)^2 (\cos^2 x) \sin x \, dx$$

$$= \int \underbrace{(1-\cos^2 x)^2 (\cos^2 x)}_{\text{FUNCTION OF}} \underbrace{\sin x \, dx}_{(-du)} = \int -(1-u^2)^2 u^2 \, du$$

$u = \cos x$

$$\Rightarrow \frac{du}{dx} = -\sin x \Rightarrow -du = \sin x \, dx$$

$$\int -(1-u^2)^2 u^2 \, du = \int -\left(1 - 2u^2 + u^4\right) u^2 \, du$$

$$= \int -u^2 + 2u^4 - u^6 \, ) \, du$$

$$= -\frac{u^3}{3} + \frac{2u^5}{5} - \frac{u^7}{7} + C$$

$$= -\frac{(\ln x)^3}{3} + 2\frac{(\ln x)^5}{5} - \frac{(\ln x)^7}{7} + C$$

$$\int \sin^n x \cos^m x \, dx$$

① $n$ ODD $\to$ KEEP ONE COPY OF $\sin x$ FOR $du = d(\cos x) = -\sin x \, dx$. REWRITE THE OTHERS USING $\sin^2 x = 1 - \cos^2 x$

② $m$ ODD $\to$ KEEP ONE COPY OF $\cos x$ FOR $du = d(\sin x) = \cos x \, dx$. REWRITE THE OTHERS USING $\cos^2 x = 1 - \sin^2 x$

③ $m, n$ EVEN ???

# (HALF/)DOUBLE-ANGLE FORMULAE !

$$\sin^2\theta = \frac{1}{2}\left(1 - \cos 2\theta\right)$$

$$\cos^2\theta = \frac{1}{2}\left(1 + \cos 2\theta\right)$$

$$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$$

$n = 2n'$

$m = 2m'$

$$\sin^n\theta \cos^m\theta$$

$$(\sin^2\theta)^{n'} (\cos^2\theta)^{m'}$$

$$\sin^4\theta \cos^2\theta$$

$$= \underline{(\sin^2\theta)}\ \underline{(\sin\theta\cos\theta)}^2$$

$n = 2$ (EVEN)
$m = 0$

**EXAMPLE 3** Evaluate $\int_0^{\pi} \sin^2 x\, dx$.

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\int_0^{\pi} \frac{1}{2}(1 - \cos 2x)\, dx = \int_0^{\pi} \frac{dx}{2} - \int_0^{\pi} \frac{\cos 2x}{2}\, dx$$

$$= \frac{1}{2}(\pi - 0) - \frac{\sin 2x}{4}\Big]_0^{\pi}$$

$$= \pi/2$$

$m = 0$

$n = 4$

**EXAMPLE 4** Find $\int \sin^4 x \, dx$.

$$\sin^2 x = \left( \frac{1 - \cos 2x}{2} \right)$$

$$\int \sin^4 x \, dx = \int (\sin^2 x)^2 \, dx = \int \left( \frac{1 - \cos 2x}{2} \right)^2 dx$$

$$= \frac{1}{4} \int (1 - 2\cos 2x + \cos^2 2x) \, dx$$

??

$$= \frac{1}{4} \left[ \boxed{\int dx} - \boxed{\int 2\cos 2x \, dx} + \boxed{\int \cos^2 2x \, dx} \right]$$

$$\int \cos^2 2x \, dx$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad (\theta = 2x)$$

$$\cos^2 2x = \frac{1+\cos(2(2x))}{2} = \frac{1+\cos 4x}{2}$$

$$\int \left(\frac{1+\cos 4x}{2}\right) dx = \int \frac{dx}{2} + \int \frac{\cos 4x}{2} \, dx$$

$$\int \sin^n x \cos^m x \, dx$$

① $n$ ODD $\to$ KEEP ONE COPY OF $\sin x$ FOR $du = d(\cos x) = -\sin x \, dx$. REWRITE THE OTHERS USING $\sin^2 x = 1 - \cos^2 x$

② $m$ ODD $\to$ KEEP ONE COPY OF $\cos x$ FOR $du = d(\sin x) = \cos x \, dx$. REWRITE THE OTHERS USING $\cos^2 x = 1 - \sin^2 x$

③ $m, n$ EVEN $\to$ USE A DOUBLE-ANGLE FORMULA / $m/2$, $n/2$