MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

(THIS WEEK) OFF HRS: T, W 4-5PM ; BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

Zoom ID:
979-4693-0650

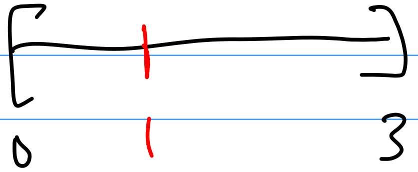
email: anuragsahay@rochester.edu

COURSE PAGE: bit.ly/sahay142

ANNOUNCEMENTS

1. WEBWORK 11 → DUE TODAY AT 11 PM
   WEBWORK 12 → " TOMORROW " "
2. FINAL EXAM ON THURSDAY (IN CLASS)
3. PLEASE FILL OUT COURSE EVALS!
4. EXTRA OFFICE HOURS : TW 4 - 5 PM
   AFTER CLASS
   BY APPOINTMENT
5. WWs ARE DUE BY AT MOST , THURSDAY NOON

BREAKOUT ROOM



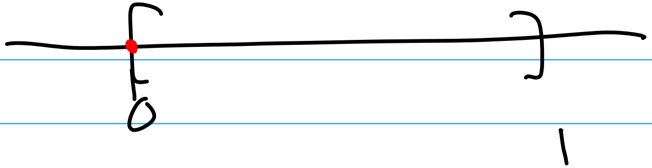
IMPROPE

EXAMPLE 7 Evaluate $\int_0^3 \frac{dx}{x-1}$ if possible.

EXAMPLE 8 Evaluate $\int_0^1 \ln x \, dx$.

$$\int_0^1 \frac{dx}{x-1} + \int_1^3 \frac{dx}{x-1}$$

$[0,1]$



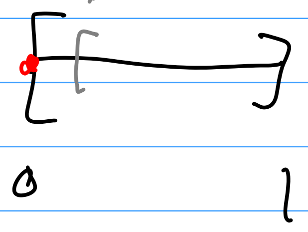
WHERE IS THE DISCONTINUITY?

$$\boxed{\lim_{t \to 0^+} \int_t^1 \ln x \, dx} \longrightarrow \text{DO THIS FIRST}$$

$t > 0$

$$\int_t^1 \ln x \, dx$$

$0^+ \leftarrow t$



$$= x \ln x - x \Big]_t^1$$

$$= (\ln 1 - 1) - (t \ln t - t)$$

$$= -1 - \underbrace{\boxed{t \ln t}}_{\to 0} \underbrace{-t}_{\to 0}$$

$0 \ln 0$

$\uparrow$

$-\infty$

$0 \times (-\infty)$

$$\boxed{\text{LOGARITHM} \ll \text{POLYNOMIAL} \ll \text{EXPONENTIAL}}$$

$$t \ln t \longrightarrow 0$$

$0 \longleftarrow t e^{-t}$ as $t \longrightarrow \infty$

$\infty \longleftarrow \dfrac{e^t}{t^2}$ as $t \longrightarrow +\infty$

$0 \longleftarrow t^3 \ln t$ as $t \longrightarrow 0$

$\infty \longleftarrow t^4 / \ln t$ as $t \longrightarrow \infty$

USE L'HÔPITAL'S RULE

$\dfrac{e^t}{\ln t}$ as $t \to \infty$

$$t \to 0$$

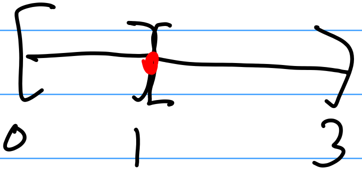
$$t \ln t = \frac{\ln t}{1/t} \qquad \ln t \to -\infty \qquad 1/t \to \infty$$

$$\lim_{t \to 0^+} t \ln t = \lim_{t \to 0^+} \frac{\frac{d \ln t}{dt}}{\frac{d(1/t)}{dt}} = \frac{1/t}{-1/t^2} = -t$$

$$\lim_{t \to 0^+} (-t) = 0$$

UNDEFINED

$$\int_0^1 \frac{dx}{x-1} + \int_1^3 \frac{dx}{x-1} \to \text{UNDEFINED} \qquad \frac{1}{x-1} \to 1$$



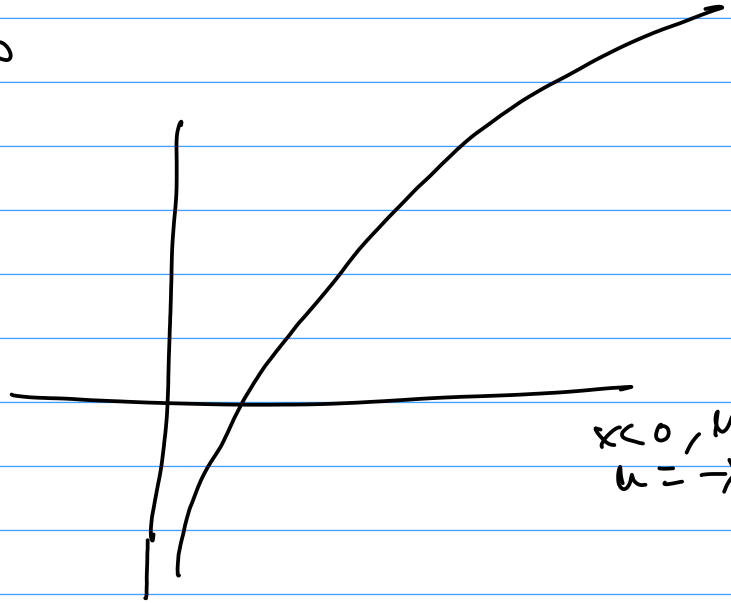
$$\int_0^1 \frac{dx}{x-1} = \lim_{t \to 1^-} \int_0^t \frac{dx}{x-1}$$

$$\int_1^3 \frac{dx}{x-1} = \lim_{t \to 1^+} \int_t^3 \frac{dx}{x-1}$$

$$\int_0^t \frac{dx}{x-1} = \ln|t-1| - \ln|0-1|$$

$$= \ln|t-1|$$

$$\ln 0 = -\infty$$



$x>0$

$$\int \frac{dx}{x} = \ln x$$

$x<0,\ u>0$

$u=-x$

$$-\int \frac{dx}{-x} = \int \frac{du}{u} = \ln u$$

$$\int_0^3 \frac{dx}{x-1} = \ln(x-1)\Big]_0^3$$

$$= \ln(3-1) - \ln(0-1)$$

$$= \ln 2$$

§ 7.5 STRATEGY FOR INTEGRATION

EXAMPLE 1 $\int \frac{\tan^3 x}{\cos^3 x}\,dx$

(i) WRITE EVERYTHING IN TERMS OF sin & cos.

$$\int \frac{\sin^3 x}{\cos^6 x}\,dx = \int \frac{(1-\cos^2 x)\sin x}{\cos^6 x}\,dx \qquad u = \cos x$$

$$\int \frac{\tan^3 x}{\cos^3 x}\, dx$$

(ii) CONVERT $\cos \to \sec$ TO REMOVE DENOM.

$$\frac{1}{\cos^3 x} = \sec^3 x$$

ODD POWER OF $\tan$

$$\int \tan^3 x \sec^3 x \, dx$$

$\Rightarrow$ SAVE $\tan x \sec x$ & REWRITE IN TERMS OF $\sec$.

EXAMPLE 2 $\int \sin\sqrt{x}\,dx = \int (\sin u)(2u\,du)$

$= \boxed{2\int u \sin u \, du}$

$u = \sqrt{x}$

$\Rightarrow \quad x = u^2$

$\Rightarrow \quad dx = 2u\,du$

INTEGRATE BY PARTS!

BREAK TILL
7:00 PM ET

EXAMPLE 3 $\displaystyle\int \frac{x^5+1}{x^3-3x^2-10x}\,dx \longrightarrow x(x-5)(x+2)$

PARTIAL FRACTIONS:

$$
\begin{array}{r|l}
 & x^2 + 3x + 19 \\
\hline
x^3 - 3x^2 - 10x & x^5 + 1 \\
 & x^5 - 3x^4 - 10x^3 \\
\hline
 & \cancel{3x^4} + \cancel{10x^3} + 1 \\
 & 19x^3 + 39x + 1
\end{array}
$$

$\frac{a}{x} + \frac{b}{x-5} + \frac{c}{x+2}$

CLEAR DENOMINATOR

$$\frac{R(x)}{x^3-3x^2-10x} = \frac{R(x)}{x(x-5)(x+2)} = \frac{a}{x} + \frac{b}{x-5} + \frac{c}{x+2}$$

CLEAR DENOM. $\longrightarrow$ MULTIPLY BY

$x(x-5)(x+2)$

$$R(x) = a(x-5)(x+2) + b(x)(x+2) + c(x)(x-5)$$

$x=-2$, $x=5$, $x=0$

EXAMPLE 4 $\int \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x \qquad du = \frac{dx}{x}$

$$\int \frac{du}{\sqrt{u}} = 2u^{1/2} + C = 2(\ln x)^{1/2} + C$$

EXAMPLE 5 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

$u = \sqrt{\frac{1-x}{1+x}}$ (RATIONALIZING)

$dx = ?$

$\int u$

$u^2 = \frac{1-x}{1+x} \Rightarrow (1+x)u^2 = 1-x$

RATIONAL FUNCTION

$\Rightarrow x = \frac{1-u^2}{1+u^2} \Rightarrow$ QUOTIENT RULE $dx = (\quad)\,du$

$$I = \int \sqrt{\frac{1-x}{1+x}}\, dx$$

$$\int \sqrt{\frac{(1-x)(1-x)}{(1+x)(1-x)}}\, dx$$

$$= \int \frac{1-x}{\sqrt{1-x^2}}\, dx = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x\, dx}{\sqrt{1-x^2}}$$

$$= \operatorname{Arcsin} x + \sqrt{1-x^2} + C$$

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \dfrac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \dfrac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
15. $\int \sinh x\,dx = \cosh x$
16. $\int \cosh x\,dx = \sinh x$
17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
18. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right), \quad a > 0$

*19. $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right|$

*20. $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

$a = 1$

STEP 1 : SIMPLIFY THE INTEGRAND

$$\int \sqrt{x}\,(1+\sqrt{x})\,dx = \int \left(\sqrt{x} + x\right)\,dx$$

$$\int \frac{\tan\theta}{\sec^2\theta}\,d\theta \xrightarrow{\sin\theta\ \&\ \cos\theta} \int \frac{\sin\theta}{\cos\theta}\cdot\cos^2\theta\,d\theta = \int \sin\theta\cos\theta\,d\theta$$

$$\int (\sin x + \cos x)^2\,dx = \int \left(\sin^2 x + \cos^2 x + 2\sin x\cos x\right)dx$$

$$= \int \left(1 + 2\sin x\cos x\right)dx$$

DOUBLE ANGLE / u - SUB

STEP 2: LOOK FOR AN OBVIOUS SUBSTITUTION OR BY PARTS.

§ 5.5 & § 7.1

WHAT IF STEP 1 + 2 ARE NOT ENOUGH?

$\int f(x)\,dx$

STEP 3: CLASSIFY THE INTEGRAND ACCORDING TO THE FORM

(a) TRIG FUNCTIONS $\rightarrow$ $f(x)$ ONLY HAS sin, cos, tan, sec, csc, cot $\Rightarrow$ $\boxed{\S\, 7.2}$

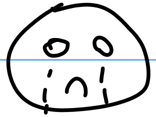
(b) RATIONAL FUNCTIONS $\rightarrow$ PARTIAL FRACTIONS $\Rightarrow$ $\boxed{\S\, 7.4}$

(c) RADICALS : (i) $\sqrt{x^2+a^2}$ OR $\sqrt{x^2-a^2}$ OR $\sqrt{a^2-x^2}$

$\Rightarrow$ TRIG SUBSTITUTION $\boxed{\S\, 7.3}$

(RATIONALIZING) $\rightarrow$ (ii) $\sqrt[n]{ax+b}$ OR $\sqrt[n]{g(x)}$ TRY $u = g(x)$ / $u^n = g(x)$

WHAT IF STEP 3 DIDN'T WORK?

STEP 4: CRY. 

THEN TRY AGAIN!

ONLY 2 ACTUAL "INTEGRATION" METHODS

↳ SUBST & ↳ BY PARTS.

TRY THEM BOTH! → MIGHT NEED INGENUITY OR INSPIRATION (OR DESPERATION!)

TRY ALSO: (i) $\boxed{\text{INGENIOUS}}$ MANIPULATIONS

e.g. $$\int \frac{dx}{1-\cos x} = \int \frac{1}{1-\cos x} \, \frac{(1+\cos x)}{(1+\cos x)} \, dx$$

$$= \int \frac{1+\cos x}{1-\cos^2 x} \, dx = \int \frac{1+\cos x}{\sin^2 x} \, dx = \int \operatorname{cosec}^2 x + \cot x \operatorname{cosec} x \, dx$$

(ii) RELATING TO PREVIOUS PROBLEMS.

(iii) TRY MORE THAN ONE METHOD!

$\boxed{\text{CAN EVERY FUNCTION BE INTEGRATED?}}$

NO!

"ELEMENTARY" → COMPOSING EXPONEN., POLYNOMIALS, + TRIG

$$\int \frac{e^x}{x}\,dx \qquad \int \sin(x^2)\,dx \qquad \int \cos(e^x)\,dx$$

$$\int \sqrt{x^3+1}\,dx \qquad \int \frac{dx}{\ln x} \qquad \int \frac{\sin x}{x}\,dx$$

$f(x)$ IS ELEMENTARY

$\Rightarrow$ $f'(x)$ IS ELEMENTARY!

$\int f(x)\, dx$ MAY NOT BE ELEMENTARY

(MOST LIKELY ELEMENTARY)