MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

(THIS WEEK) OFF HRS: T, W 4-5PM ; BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

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COURSE PAGE: bit.ly/sahay142

ANNOUNCEMENTS

1. WEBWORK 12 → TODAY AT 11 PM
2. FINAL EXAM ON THURSDAY (IN CLASS)
3. PLEASE FILL OUT COURSE EVALS !
4. EXTRA OFFICE HOURS TOMORROW MORNING.
5. WW → FINAL DEADLINE IS TOMORROW NOON.

5. WWS ARE DUE BY AT MOST, THURSDAY NOON

$$\int_0^{2\pi} (\sin x)^4 (\cos x)^2 \, dx$$

① $\left\{ \begin{array}{ll} \sin^2\theta & = \frac{1}{2}(1 - \cos 2\theta) \\ \cos^2\theta & = \frac{1}{2}(1 + \cos 2\theta) \\ \sin\theta\cos\theta & = \frac{1}{2}(\sin 2\theta) \end{array} \right.$ ②

$$(\sin x \cos x)^2 \sin^2 x$$

$$= \left(\frac{\sin 2x}{2}\right)^2 \cdot \left(\frac{1 - \cos 2x}{2}\right)$$

$$(\sin^2 x)^2 = \left[\frac{1}{2}(1 - \cos 2x)\right]^2$$

$$(\cos x)^2 = \frac{1}{2}(1 + \cos x)$$

$$\int_0^{2\pi} \left(\frac{\sin 2x}{2}\right)^2 \cdot \left(1 - \frac{\cos 2x}{2}\right) \, dx$$

$$\frac{1}{8} \int_0^{2\pi} \left(\sin^2 2x\right) - \left(\sin^2 2x \cos 2x\right) dx$$

$$= \frac{1}{8} \int_0^{2\pi} \sin^2 2x \, dx - \frac{1}{8} \int_0^{2\pi} \sin^2 2x \left(\cos 2x \, dx\right)$$

$u = \sin 2x$

$$\frac{1}{8}\int_0^{2\pi} \underbrace{\sin^2 2x}_{u^2}\,\overbrace{\left(\cos 2x \; dx\right)}^{du/2}$$

$$u = \sin 2x$$

$$\Rightarrow du = 2\cos 2x \; dx$$

$$\frac{1}{8}\int_0^0 \frac{u^2}{2}\,du = 0$$

$$u(0) = \sin 2(0) = 0$$

$$u(2\pi) = \sin 2(2\pi) = 0$$

$$\frac{1}{8}\int_0^{2\pi} \sin^2 2x \; dx$$

$(\sin^2 \quad \cos^0)$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$= \frac{1}{8}\int_0^{2\pi} \frac{1 - \cos 4x}{2} \; dx$$

$\theta = 2x$

$$= \frac{1}{16}\left[\int_0^{2\pi} dx - \int_0^{2\pi} \cos 4x \; dx\right]$$

$$= \frac{1}{16}\left[2\pi\right] = \frac{\pi}{8}$$

$$\left.\frac{\sin 4x}{4}\right]_0^{2\pi} = \frac{\sin 8\pi - \sin 0}{4} = 0$$

$$\int_1^\infty \frac{e^x}{x}\,dx \nearrow \text{DIVERGENT.}$$

IMPORTANT

COMPARISON TEST

$x \geq 1$

$$\frac{e^x}{x} \geq \frac{1}{x} \boxed{\geq 0}$$

$e^x \geq 1$ FOR $x \geq 0$

$$+\infty = \int_1^\infty \frac{e^x}{x}\,dx \geq \int_1^\infty \frac{dx}{x} = +\infty$$

$$\int_1^{\infty} \frac{dx}{x} = \lim_{t \to \infty} \int_1^{t} \frac{dx}{x}$$

$$= \lim_{t \to \infty} \Big[\ln x \Big]_1^{t}$$

$$= \lim_{t \to \infty} \Big[\ln t \Big]$$

$$= +\infty \rightarrow \text{DIVERGENT}$$

BREAK TILL

6:48 PM ET

$$\int \frac{\sin 2\theta}{(1-\sin\theta)^2\,(1+\sin^2\theta)}\,d\theta$$

DOUBLE ANGLE FORMULA

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\int \frac{2\overbrace{\sin\theta}^{u}\ \overbrace{\cos\theta\ d\theta}^{du}}{\underbrace{(1-\sin\theta)^2}_{(1-u)^2}\ \underbrace{(1+\sin^2\theta)}_{(1+u^2)}}$$

$u = \sin\theta \Rightarrow du = \cos\theta\, d\theta$

$$\int \frac{2u \quad du}{(1-u)^2(1+u^2)}$$

$$\frac{2u}{(1-u)^2(1+u^2)} = \frac{a}{(1-u)^2} + \frac{b}{(1-u)} + \frac{cu+d}{u^2+1}$$

$$\Rightarrow \quad 2u = a(u^2+1) + b(1-u)(u^2+1) + (cu+d)(1-2u+u^2)$$

$$0 = a + b + d$$
$$2 = -b + c - 2d$$
$$0 = a + b + d - 2c$$
$$0 = -b + c$$

$u=1$,

$2a = 2$

$a = 1$

$$0 = 1 + b - 1$$

$$0 = 1 + (-1) - c$$

$$0 = -b + c$$

$$c = 0$$

$$b = 0$$

$u = 1$, $2a = 2$

$a = 1$

$d = -1$

$a = 1$, $b = c = 0$,

$d = -1$

3 TECHNIQUES:

1. DOUBLE-ANGLE
2. u-SUBSTITUTION
3. PARTIAL FRACTION

$$\int \frac{2u \, du}{(1-u)^2(1+u^2)} = \int \frac{du}{(u-1)^2} - \int \frac{du}{u^2+1} = \frac{-1}{u-1} - \arctan u + C$$

$$= \frac{-1}{\sin\theta - 1} - \arctan(\sin\theta) + C$$

$$\int \sin(\ln x)\, dx$$

1. INTEGRATION BY PARTS

$$u = \sin(\ln x) \Rightarrow du = \frac{\cos(\ln x)}{x}\, dx$$

$$dv = 1 \longrightarrow v = x$$

$$x \sin(\ln x) - \int \cancel{x} \left(\frac{\cos(\ln x)}{\cancel{x}} \right) dx$$

$$x \sin(\ln x) - \int \cos(\ln x)\, dx$$

$$u = \cos(\ln x) \Rightarrow du = \frac{-\sin(\ln x)}{x}\, dx$$

$$dv = dx \Rightarrow v = x$$

$$x \sin(\ln x) - \left[x \cos(\ln x) - \int \cancel{x} \left(- \frac{\sin \ln x}{\cancel{x}} \right) dx \right]$$

$$\overbrace{\int \sin(\ln x)\, dx}^{I} = x \sin(\ln x) - x \cos \ln x - \overbrace{\int \sin(\ln x)\, dx}^{-I}$$

$$\Rightarrow 2I = x \sin(\ln) - x \cos(\ln x)$$

$$I = \frac{1}{2}\left[x \sin(\ln x) - x \cos(\ln x)\right] + C$$

$$\int \underbrace{\sin(\ln x)}_{\sin u}\, dx = \int (\sin u)(e^u\, du)$$

$$u = \ln x \iff x = e^u$$

$$dx = e^u\, du$$

$$\int e^u \sin u\, du$$

$$\int \frac{(e^u)}{\sqrt{3-2e^u-e^{2u}}} \cdot \underbrace{(e^u\,du)}_{d(e^u)}$$

$$\boxed{x = e^u} \Rightarrow dx = e^u\,du$$

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$$

COMPLETE THE SQUARE

$\sqrt{q(x)}$

$q \to$ QUADRATIC

COMPLETE THE SQUARE

& DO A SUBSTITUTION

$\sqrt{a^2 - x^2}$

$\sqrt{x^2 - a^2}$

$\sqrt{x^2 + a^2}$

TRIG SUBSTITUTION

$$3 \underbrace{-2x - x^2} = 4 - 1 - 2x - x^2 = 4 - (1+x)^2$$

$$-1 - 2x - x^2$$

$$= -(1 + 2x + x^2)$$

$$= -(1+x)^2$$

$\left(\dfrac{\text{COEFF. OF } x}{2 \times \text{COEFF OF } x^2} \right)$

$1 + \tan^2\theta = \sec^2\theta$

$\sqrt{a^2 + x^2}$

$\sqrt{a^2 - x^2} \to 1 - \sin^2\theta = \cos^2\theta$

$\sqrt{x^2 - a^2} \longrightarrow \sec^2\theta - 1 = \tan^2\theta$

$$\int \frac{x}{\sqrt{4 - (x+1)^2}} \, dx$$

$v = x + 1$

$v = 2\sin\theta$

$\sqrt{4 - v^2}$

$x + 1 = 2\sin\theta \Rightarrow dx = 2\cos\theta \, d\theta$

$x = 2\sin\theta - 1$

$$\sqrt{4-(x+1)^2} = 2\cos\theta$$

$$\int \frac{(2\sin\theta - 1)(2\cos\theta \, d\theta)}{2\cos\theta}$$

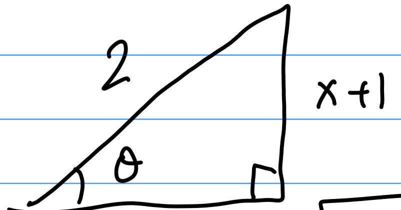
$$= \int 2\sin\theta - \int d\theta$$

$$= -2\cos\theta - \theta + C$$

$$x+1 = 2\sin\theta$$

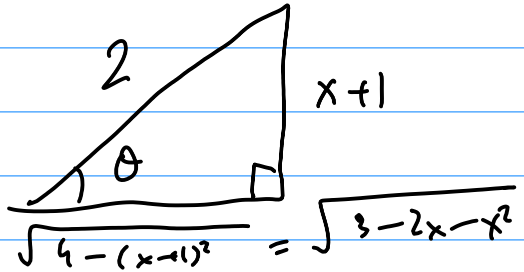
$$\Rightarrow \theta = \arcsin\left(\frac{x+1}{2}\right)$$

$$-\arcsin\left(\frac{x+1}{2}\right) + C$$



$$\sqrt{4-(x+1)^2} = \sqrt{3-2x-x^2}$$

$$\sin\theta = \frac{x+1}{2}$$



$$\tan\theta = \frac{\sqrt{3-2x-x^2}}{2}$$

$$-\sqrt{3-2x-x^2} - \arcsin\left(\frac{x+1}{2}\right) + C$$

$x = e^u$

$$= -\sqrt{3-2e^u - e^{2u}} - \arcsin\left(\frac{e^u+1}{2}\right) + C$$