MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

OFF HRS: M, F 4-5PM ;
BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

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COURSE PAGE : bit.ly/sahay142

ANNOUNCEMENTS

1. WEBWORK DEADLINES: (a) WW 3 → TODAY
   (b) WW 4 → FRIDAY
2. WILL CHANGE EXAM SYLLABUS TO INCLUDE § 5.4
3. EXAM REVIEW TOMORROW. LOOK OVER PRACTICE EXAMS.
4. PARTICIPATION GRADES ARE UP.
5. CHECK LAST EMAIL!

§ 5.4 INDEFINITE INTEGRALS AND THE NET CHANGE THEOREM

(CONTD.)

NET CHANGE THEOREM
(A.K.A. FTC2 FOR PROFESSIONALS)

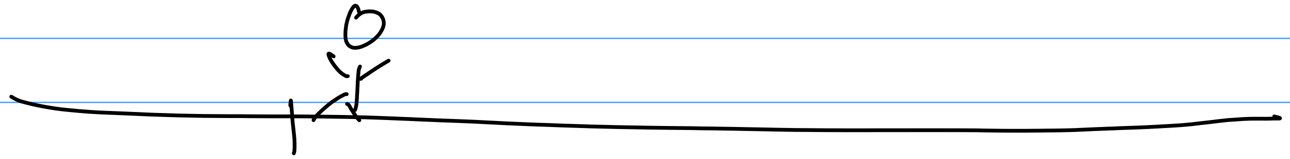
THM: "THE INTEGRAL OF A RATE OF CHANGE IS THE **NET** CHANGE"

$$\int_a^b F'(t)\, dt = F(b) - F(a)$$

* APPLIES TO ANY RATE OF CHANGE

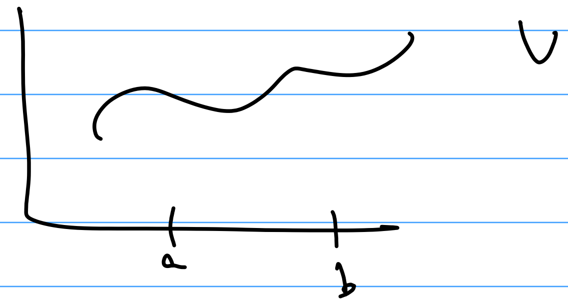
DISPLACEMENT & DISTANCE

$v = v(t)$



$t = a$ $\qquad\qquad$ $t = b$

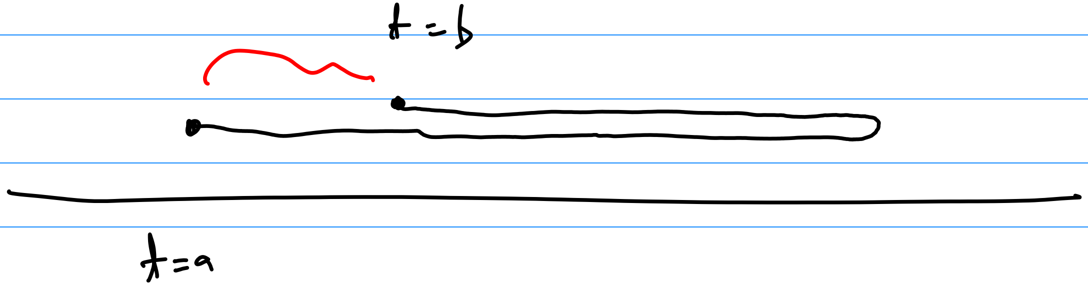
$v(t)$



$$\int_a^b v(t)\,dt = s(b) - s(a)$$

$s \longrightarrow$ DISPLACEMENT

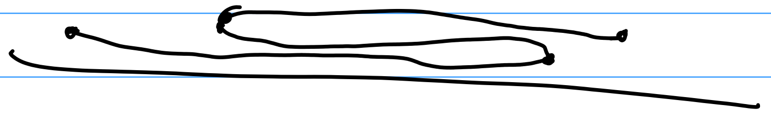
$$\frac{ds}{dt} = v$$



$$s(b) - s(a)$$

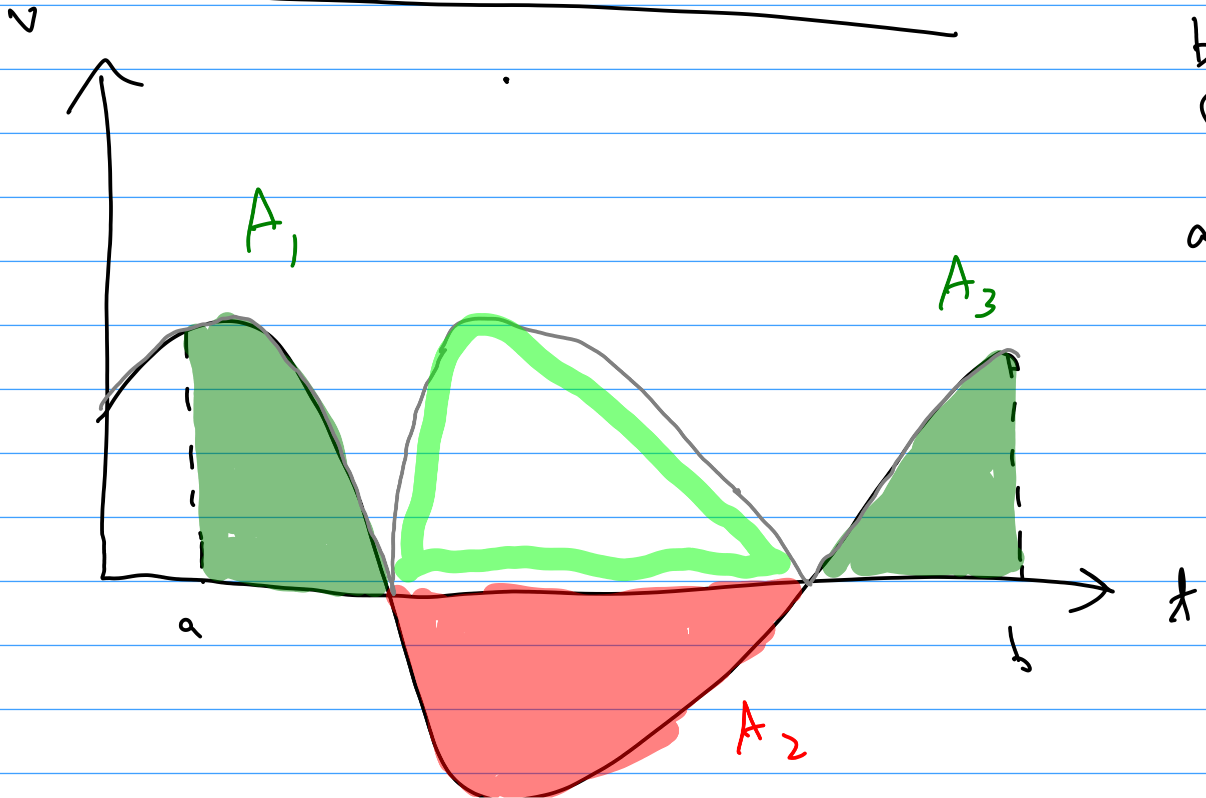
$$= \int_a^b v(t)\,dt$$

DISTANCE TRAVELLED $= \int_a^b |v(t)|\,dt$



$$\int_a^b v(t)\,dt = A_1 - A_2 + A_3$$

$$\int_a^b |v(t)|\,dt = A_1 + A_2 + A_3$$



$|v(t)| \rightarrow$ SPEED

EXAMPLE 6 A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

(a) Find the displacement of the particle during the time period $1 \leqslant t \leqslant 4$.

(b) Find the distance traveled during this time period.

$$\rightarrow v(t) = t^2 - t - 6$$

-9.5

$$\text{DISPLACEMENT} = \int_1^4 v(t)\,dt = \int_1^4 (t^2 - t - 6)\,dt$$

-9.5

$\parallel$

$$= \left.\frac{t^3}{3} - \frac{t^2}{2} - 6t\,\right]_1^4 = \frac{4^3}{3} - \frac{4^2}{2} - 6\cdot 4 - \frac{1}{3} + \frac{1}{2} + 6$$

EXAMPLE 6 A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

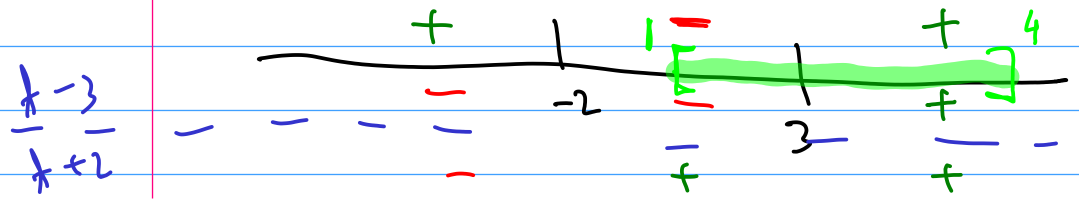
(a) Find the displacement of the particle during the time period $1 \leqslant t \leqslant 4$.

(b) Find the $\boxed{\text{distance}}$ traveled during this time period.

$$\text{DISTANCE} = \int_1^4 |v(t)|\,dt = \int_1^4 |t^2 - t - 6|\,dt$$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$t^2 - t - 6 = (t-3)(t+2)$$



$v(t) < 0$ $\qquad\qquad$ $v(t) > 0$

$$\int_1^4 |v(t)|\,dt = \int_1^3 |v(t)|\,dt + \int_3^4 |v(t)|\,dt$$

$$= \int_1^3 \left[-v(t)\right] dt + \int_3^4 v(t)\,dt$$

$$= \int_1^3 -(t^2 - t - 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt$$

$$\int_1^3 -(t^2 - t - 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt$$

$$= \int_1^3 (-t^2 + t + 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt$$

$$= \left[-\frac{t^3}{3} + \frac{t^2}{2} + 6t\right]_1^3 + \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_3^4$$

$$= \left[-\frac{3^3}{3} + \frac{3^2}{2} + 6\cdot 3\right] - \left[\frac{-1^3}{3} + \frac{1^2}{2} + 6\cdot 1\right] + \left[\frac{4^3}{3} - \frac{4^2}{2} - 6\cdot 4\right] - \left[\frac{3^3}{3} - \frac{3^2}{2} - 6\cdot 3\right]$$

§ 5.5 THE SUBSTITUTION RULE

(AKA CHAIN RULE REVERSED)

* ALSO CALLED u- SUBSTITUTION

ANATOMY OF INTEGRAL

$$\int_a^b f(x)\, dx$$

WHY IS THIS THERE?

$\boxed{\sum f(x) \,/\, \int f(x)}$

REVIEW : CHAIN RULE

$$\left[\underbrace{\frac{d}{dx} f(g(x))}_{dy/dx} = (f \circ g)'(x) = \underbrace{g'(x)}_{\frac{du}{dx}} \cdot \underbrace{f'(g(x))}_{\frac{dy}{du}} \right]$$

$$\frac{dy}{du} = f'(u)$$

$$y = f(g(x)) \qquad u = g(x) \qquad y = f(u)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

HEURISTIC $\longrightarrow$ $u = g(x)$

$$\Rightarrow \frac{du}{dx} = g'(x)$$

$$\Rightarrow \boxed{du = g'(x)\, dx}$$

????

e.g.

$$\int 2x \underbrace{\sqrt{1+x^2}}_{\sqrt{u}} dx$$

$$u = 1 + x^2$$

$$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$$

$$\int 2x\sqrt{1+x^2}\,dx = \int \underbrace{\left(\sqrt{1+x^2}\right)}_{\sqrt{u}} \underbrace{(2x\,dx)}_{du} = \int \sqrt{u}\,du = \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2u^{3/2}}{3} + C$$

$$\int 2x\sqrt{1+x^2}\,dx = \frac{2}{3}\cdot u^{3/2} + C$$

$u = 1+x^2$

$$= \frac{2}{3}\left(1+x^2\right)^{3/2} + C$$

LET'S CHECK:

$v = 1+x^2$

$$\frac{d}{dx}\left[\frac{2}{3}\left(1+x^2\right)^{3/2}\right] = \underbrace{\frac{d}{dx}\left(1+x^2\right)}_{du/dx} \cdot \underbrace{\frac{d}{dv}\left(\frac{2}{3}v^{3/2}\right)}_{\frac{dy}{dx}}$$

$v = 1 + x^2$

$$\underbrace{\frac{d}{dx}\left(1 + x^2\right)}\cdot\underbrace{\frac{d}{dv}\left(\frac{2}{3}v^{3/2}\right)}$$

$$= (2x)\left[\frac{2}{3}\cdot\frac{3}{2}v^{3/2-1}\right]$$

$$= (2x)\cdot v^{1/2} = 2x\sqrt{1 + x^2}$$

e.g.

$$\int x^3 \underbrace{\cos(x^4+2)}_{\cos u}\, dx$$

$v = \cos(x^4+2)$

$\frac{dv}{dx} = 4x^3 \cdot -\sin(x^4+2)$

$dv = 4x^3 \, -\sin(x^4+2) \, dx$

$$u = x^4 + 2$$

$$\frac{du}{dx} = 4x^3 \Rightarrow \boxed{du = 4x^3\, dx} \longrightarrow \frac{du}{4} = x^3 dx$$

$$\int x^3 \cos(x^4+2)\, dx = \int \cos(x^4+2) \cdot (x^3 dx)$$

$$\int \cos u \left(\frac{du}{4}\right) = \frac{1}{4}\int \cos u \, du$$

$$\frac{1}{4}\int \cos u \, du = \frac{\sin u}{4} + C$$

$(u = x^4 + 2)$

$$\int x^3 \cos(x^4+2)\, dx = \frac{\sin(x^4+2)}{4} + C$$

THE RULE

IF $u = g(x)$ IS DIFFERENTIABLE, THEN

$$\int \underbrace{f(g(x))}_{f(u)} \underbrace{g'(x)\,dx}_{du} = \int f(u)\,du$$

$$\frac{du}{dx} = \frac{d}{dx} g(x) = g'(x)$$

$$du = g'(x)\,dx$$

LEIBNIZ NOTATION:

$$\int \left[f(u(x)) \frac{du}{dx} \right] dx = \int f(u)\,du$$

Pf INTEGRATE THE CHAIN RULE

$F' \longrightarrow f$

$$F'(g(x))\, g'(x) = \frac{d}{dx}\left(F(g(x))\right)$$

$$\int f(g(x))\, g'(x)) = F(g(x))$$

$$= \int f(u)\, du$$

$\boxed{\text{BOOK}}$

EXAMPLE 2 Evaluate $\int \sqrt{2x+1}\,dx$.

$$\int \underbrace{\left(\tfrac{1}{2}\sqrt{2x+1}\right)}_{f(g(x))} \underbrace{(2dx)}_{g'(x)dx} = \int f(u)\,du$$

$$g(x) = 2x+1$$

$$f(x) = \frac{1}{2}\sqrt{x}$$

$$u = 2x+1$$

$$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$$

$$\int \sqrt{u}\left(\frac{du}{2}\right) = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right] + C$$

$$= \frac{u^{3/2}}{3} + C$$

$$= (2x+1)^{3/2}/3 + C$$

BREAKOUT ROOM 1

$u = 1 - 4x^2$

$\boxed{du = -8x\,dx}$

$\frac{du}{-8}$

EXAMPLE 3 Find $\int \frac{x}{\sqrt{1-4x^2}}\,dx$.

$-\frac{1}{4}(1-4x^2)^{1/2} + C$

$\int \frac{}{\sqrt{u}}$

$\sqrt{u}$

EXAMPLE 4 Evaluate $\int e^{5x}\,dx$.

$\int \left(\frac{1}{\sqrt{u}}\right)\left(\frac{du}{-8}\right) = -\frac{1}{8}\int \frac{du}{u^{1/2}}$

$u = 1 - 4x^2$

$$-\frac{1}{8}\int \left(\frac{1}{u^{1/2}}\right) du = -\frac{1}{8}\int u^{-1/2}\, du = -\left(\frac{1}{8}\right)\frac{u^{1/2}}{1/2} + c$$

$$= -\frac{1}{8}\, 2u^{1/2} + C$$

$$= -\frac{2\left(1-4x^2\right)^{1/2}}{8} + C$$

$$= -\frac{1}{4}\left(1-4x^2\right)^{1/2} + C$$

BREAK TILL

7:00 PM EST

$\int c\, f(x)\,dx = c \int f(x)\,dx$

BREAKOUT ROOM 2

$u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5}$

EXAMPLE 4 Evaluate $\int e^{5x}\,dx$.

$5e^{5x} + 5 + C$

$\int e^u \frac{du}{5} = \frac{1}{5}\int e^u\,du$

$= \frac{e^u}{5} + C$

$= \frac{e^{5x}}{5} + C$

EXAMPLE 5 Find $\int \sqrt{1 + x^2}\, x^5\,dx$.

$\left(e^u + \frac{1}{5}\right)$

$$\int \underbrace{\sqrt{1+x^2}}_{\sqrt{u}} \; \underbrace{x^5 \, dx} \rightarrow x^4 \cdot \frac{du}{2}$$

x^4 = IN TERMS OF u

$$x^4 = (x^2)^2 = (u-1)^2$$

1. FIND THE RIGHT SUBSTITUTION

$$\boxed{u = 1+x^2}$$

2. FIND $\frac{du}{dx}$ $\Rightarrow$ $\frac{du}{dx} = 2x$

3. CLEAR DENOMINATORS OF dx $\quad du = 2x\,dx$

4. DO THE SUBSTIT.

$$\int (\sqrt{u}) \; (u-1)^2 \, du$$

$$\int (\sqrt{u}) \quad (u-1)^2 \left(\frac{du}{2}\right)$$

$\sqrt{u} = u^{1/2}$

$u^a \cdot u^b = u^{a+b}$

$$= \int (\sqrt{u}) (\quad u^2 - 2u + 1) \left(\frac{du}{2}\right)$$

$$= \int \frac{\left(u^{5/2} - 2u^{3/2} + u^{1/2}\right)}{2} \, du$$

$$= \frac{1}{2}\left[\frac{u^{7/2}}{7/2} - \frac{2u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} \right] + C$$

$$= \frac{u^{7/2}}{7} - \frac{2u^{5/2}}{5} + \frac{u^{3/2}}{3} + C = \frac{(1+x^2)^{7/2}}{7} - \frac{2(1+x^2)^{5/2}}{5} + \frac{(1+x^2)^{3/2}}{3} + C$$

$$\int f(g(x))\, g'(x)\, dx$$

u-SUBST. FOR INDEFINITE INTEGRALS $\rightsquigarrow$ FTC

SUBSTITUTION FOR DEFINITE INTEGRALS

IF g' IS CONTINUOUS ON $[a, b]$
AND f IS CONTINUOUS ON THE RANGE
OF g, THEN

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

EXAMPLE 7 Evaluate $\int_0^4 \sqrt{2x+1}\, dx$ $= \frac{26}{3}$

$$\int_0^4 \sqrt{2x+1}\, dx$$

$u = 2x + 1$

$du = 2dx$

$u(0) = 2 \times 0 + 1 = 1$

$u(4) = 2 \times 4 + 1 = 9$

$$\int_1^9 \sqrt{u} \left(\frac{du}{2}\right) = \int_1^9 \frac{\sqrt{u}}{2}\, du$$

3

$$\int_1^9 \frac{\sqrt{u}}{2}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_1^9$$

$$= \frac{1}{2}\left[\frac{9^{3/2}}{3/2} - \frac{1^{3/2}}{3/2}\right]$$

$9 = 3^2$

$$= \left[\frac{9^{3/2}}{3} - \frac{1}{3}\right]$$

$$= \left[\frac{3^3}{3} - \frac{1}{3}\right] = 9 - \frac{1}{3} = \frac{26}{3}$$

EXAMPLE 9 Evaluate $\int_1^e \frac{\ln x}{x}\,dx$. $= \int_0^1 u\,du = \frac{u^2}{2}\Big]_0^1 = \frac{1^2}{2} - \frac{0^2}{2}$

$= 1/2$

$$\int_1^e \frac{\ln x}{x}\,dx$$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$$\int \frac{u}{x}(x\,du) = \int_0^1 u\,du$$

$du = \frac{dx}{x}$

$dx = x\,du$

$u = \ln x$

$x = 1 \quad u = \ln 1 = 0, \quad x = e \quad u = \ln e = 1$