

MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

OFF HRS: M, F 4-5PM;
BY APPOINTMENT

LECTURES:

5:45 PM - 7:50 PM (ET)
M, T, W, R

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COURSE PAGE : bit.ly/sahay142

ANNOUNCEMENTS

1. WEBWORK DEADLINES :
 - (a) WW 5 → WEDNESDAY, 11 PM
 - (b) WW 6 → FRIDAY

(WW 6 TO BE RELEASED)
2. PARTICIPATION POINTS FOR LAST WEEK WILL BE UPDATED WITH EXAM SCORES.
3. IMP: WEBWORK DEADLINES ARE FLEXIBLE.

u-SUBSTITUTION

$$\int f(\overbrace{g(x)}) \underbrace{g'(x)} dx = \int f(\overbrace{u}) du$$

1. FIND THE RIGHT SUBSTITUTION ($u = g(x)$)

2. FIND $\frac{du}{dx} = g'(x)$

$$du = g'(x) dx$$

3. CLEAR DENOMINATORS OF dx $du = g'(x) dx$

4. DO THE SUBSTIT. (PLUG IN $dx = \frac{du}{g'(x)}$)
& $u = g(x)$

HOW TO CHOOSE u ?

1. USE YOUR CHAIN RULE INTUITION
2. [LOOK FOR COMPLEX FUNCTIONS COMPOSED TOGETHER
3. [LOOK FOR DERIVATIVES OF FUNCTIONS YOU RECOGNIZE.

$$\int \underbrace{\ln x}_{g'(x)} \underbrace{[2x^2 - 1]}_{g(x) = 2x^2 - 1} dx = \int (2u^2 - 1) du$$

$u = \ln x$ $u = 2x^2 - 1$ $\frac{d}{dx}$

$$\left(\int \ln x^3 \right) \xrightarrow{f(g(x))} f(g(x))$$

$g(x) = x^3$
 $f(x) = \ln x$

$$35. \int \frac{(\arctan x)^2}{x^2 + 1} dx \rightarrow du$$

$$u = \arctan x$$

$$1. (\arctan x)^2 \quad [\text{CHAIN RULE}]$$

$$2. \frac{1}{x^2 + 1} = \frac{d}{dx} (\arctan x)$$

$$\int \frac{(\arctan x)^2}{x^2 + 1} dx \rightarrow du$$

$$1. u = \arctan x \quad (\text{FIND } u)$$

$$2. \frac{du}{dx} = \frac{1}{1+x^2} \quad (\text{FIND } \frac{du}{dx})$$

$$3. du = \frac{dx}{1+x^2}$$

$$4. \int u^2 du$$

$$5. \frac{u^3}{3} + C = \frac{(\arctan x)^3}{3} + C$$

CHECK

$$\frac{d}{dx} \left[\frac{(\arctan x)^3}{3} + C \right]$$

$$\frac{d}{dx} \left(\frac{v^3}{3} + C \right) = \frac{3v^2}{3} = v^2$$

$$\frac{dv}{dx} \cdot \frac{d}{dv} \left(\frac{v^3}{3} + C \right)$$

$$v = \arctan x$$

$$= \left(\frac{1}{1+x^2} \right) (v^2)$$

$$\frac{dv}{dx} = \frac{1}{1+x^2}$$

$$= \frac{(\arctan x)^2}{1+x^2}$$



66. $\int_0^1 \frac{e^x}{1 + e^{2x}} dx$

Annotations:
 e^x is circled with an arrow pointing to $g(x)$ (if $g(x) = e^x$)
 e^{2x} is circled with an arrow pointing to $(e^x)^2$

IIT:
 $\frac{a^{bc}}{a} = (a^c)^b$

$u = e^x$

1. $u = e^x$

2. $\frac{du}{dx} = e^x$

3. $du = e^x dx$

$\int_0^1 \frac{e^x dx}{1 + e^{2x}} = \int_1^e \frac{du}{1 + u^2}$

Annotations:
 $e^x dx$ is boxed with du above it.
 e^{2x} is boxed with u^2 below it.
 1 is written below the integral sign on the right.

$u|_{x=0} = e^0 = 1$

$u|_{x=1} = e^1 = e$

(DEFINITE INTEGRALS) 5. FIND THE RIGHT LIMITS OF INT.

$x=0$ to $x=1$

$u=1$ to $u=e$

$$\int_1^e \frac{1}{1+u^2} du = \left[\arctan u \right]_1^e$$
$$= \arctan e - \arctan 1$$

$$64. \int_1^4 \frac{\sqrt{2 + \sqrt{x}}}{\sqrt{x}} dx = \int \sqrt{u} du$$

The integral is annotated with a red circle around the numerator $\sqrt{2 + \sqrt{x}}$ and a blue circle around the denominator \sqrt{x} . An arrow points from the red circle to the label \sqrt{u} above it. Another arrow points from the blue circle to the label du below it.

1. $u = 2 + \sqrt{x}$

2. $\frac{du}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$

$$2du = \frac{dx}{\sqrt{x}}$$