# MATH 142 (SUMMER '21, SESH A2)

ANURAG SAHAY

OFF HRS: M, F 4-5PM ; BY APPOINTMENT

LECTURES:
5:45 PM – 7:50 PM (ET)
M, T, W, R

Zoom ID:
979-4693-0650

email: anuragsahay@rochester.edu

COURSE PAGE: bit.ly/sahay142

# ANNOUNCEMENTS

1. WEBWORK DEADLINES : (a) WW 5 → WEDNESDAY, 11 PM
   (b) WW 6 → FRIDAY
   (WW 6 TO BE RELEASED)

2. PARTICIPATION POINTS FOR LAST WEEK WILL BE UPDATED WITH EXAM SCORES.

3. IMP : WEBWORK DEADLINES ARE FLEXIBLE.

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

## u- SUBSTITUTION

1. FIND THE RIGHT SUBSTITUTION $(u = g(x))$

2. FIND $\frac{du}{dx} = g'(x)$ $\boxed{du = g'(x)dx}$

3. CLEAR DENOMINATORS OF dx $du = g'(x)dx$

4. DO THE SUBST IT. ( PLUG IN $dx = \frac{du}{g'(x)}$ & $u = g(x)$ )

# HOW TO CHOOSE u?

1. USE YOUR CHAIN RULE INTUITION
2. LOOK FOR COMPLEX FUNCTIONS COMPOSED TOGETHER
3. LOOK FOR DERIVATIVES OF FUNCTIONS YOU RECOGNIZE.

$g(x) = \sin x$

$$\int \overbrace{\cos x}^{g'(x)} \left[2\sin^2 x - 1\right] dx = \int (2u^2 - 1)\, du$$

$u = \sin x$ $\quad du = \cos x \, dx$

$$\frac{d}{dx}\left(\tan x^3\right) \qquad f(g(x))$$

$g(x) = x^3$

$f(x) = \tan x$

**35.** $\displaystyle\int \frac{(\arctan x)^2}{x^2+1}\,dx \rightarrow du$

$u = \text{Arctan}\, x$

1. $(\text{Arctan}\, x)^2$ [ CHAIN RULE ]

2. $\dfrac{1}{x^2+1} = \dfrac{d}{dx}(\text{Arctan}\, x)$

$\displaystyle\int \frac{(\text{Arctan}\ x)^2}{x^2+1}\,dx$

$(\text{Arctan}\ x)^2 \rightarrow u^2$

$\dfrac{dx}{x^2+1} \rightarrow du$

1. $u = \text{Arctan}\ x$ (FIND $u$)

2. $\dfrac{du}{dx} = \dfrac{1}{1+x^2}$ (FIND $\dfrac{du}{dx}$)

3. $du = \dfrac{dx}{1+x^2}$

4. $\displaystyle\int u^2\,du$

5. $\dfrac{u^3}{3} + C = \dfrac{(\text{Arctan}\, x)^3}{3} + C$

CHECK

$$\frac{d}{dx}\left[\frac{(\arctan x)^3}{3} + C\right]$$

$$\frac{d}{d}\left(\frac{v^3}{3} + C\right) = \frac{3v^2}{3} = v^2$$

$$\frac{dv}{dx} \cdot \frac{d\left(v^3/3 + C\right)}{dv}$$

$$v = \arctan x$$

$$\frac{dv}{dx} = \frac{1}{1+x^2}$$

$$= \left(\frac{1}{1+x^2}\right)\left(v^2\right)$$

$$= \frac{(\arctan x)^2}{1+x^2}$$

$\boxtimes$

HINT: $a^{bc} = (a^c)^b$

**66.** $\displaystyle\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$

$e^x \to g'(x)$ (if $g(x)=e^x$)

$e^{2x} \to (e^x)^2$

$u = e^x$

1. $u = e^x$

2. $\dfrac{du}{dx} = e^x$

3. $du = e^x\,dx$

(DEFINITE INTEGRAL) 5. FIND THE RIGHT LIMITS OF INT.

$$\int_0^1 \frac{\overbrace{\boxed{e^x\,dx}}^{du}}{1+\underbrace{\boxed{e^{2x}}}_{u^2}} = \int_1^e \frac{du}{1+u^2}$$

$x=0$ to $x=1$

$u=1$ $u=e$

$u\big|_{x=0} = e^0 = 1$

$u\big|_{x=1} = e^1 = e$

$$\int_1^e \frac{1}{1+u^2}\,du = \arctan u \Big]_1^e$$

$$= \arctan e - \arctan 1$$

$\sqrt{u}$

**64.** $\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}\,dx = \int \sqrt{u}\,du$

$du$

1. $u = 2 + \sqrt{x}$

2. $\frac{du}{dx} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

$2du = \frac{dx}{\sqrt{x}}$