Math 142: Calculus II

Midterm 1

June 7th, 2021

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

- The exam will be 75 minutes long. You will get extra time in the end to upload the exam to Gradescope.
- There are 12 pages.
- A formula sheet is provided.
- No calculators, phones, electronic devices, books, notes are allowed during the exam. The only materials you are allowed to use are are pen/pencil and paper. In particular, you are NOT allowed to take the exam on a tablet.
- You are allowed to use a phone or tablet to take photographs of your answer sheet once the exam is over. If you finish early, you must take permission before taking photographs. Once you start taking photographs, you are not allowed to write.
- **Show all work and justify all answers as much as possible.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Numerical or algebraic simplifications of answers are not required, unless specifically stated otherwise.

QUESTION	VALUE	SCORE
1	0	
2	20	
3	15	
4	15	
5	18	
6	12	
7	20	
8	20	
9	20	
TOTAL	140	

Formulas

- Area of a circle of radius r, $= \pi r^2$.
- Area of the curved part of a circular cylinder of radius r and height $h = 2\pi r h$
- $\sin^2 x + \cos^2 x = 1$.
- $\sec^2 x = 1 + \tan^2 x$.
- Formula for the left-endpoint Riemann sum for $\int_a^b f(x)\,dx$ is given by

$$\frac{b-a}{n}\sum_{j=0}^{n-1} f\left(a + \frac{j(b-a)}{n}\right).$$

- Formula for the right-endpoint Riemann sum for $\int_a^b f(x)\,dx$ is given by

$$\frac{b-a}{n}\sum_{j=1}^{n} f\left(a + \frac{j(b-a)}{n}\right).$$

1. (0 points) Copy the following honesty pledge on to your answer sheet. Remember to sign and date it.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

2. (20 points)

(a) Find the vertical and horizontal asymptotes of

$$g(x) = \frac{9x^3}{(x-1)(x^2-5x+6)}.$$

(b) Does the following function have any symmetry? If so, what kind?

$$h(x) = e^{x^2} + \frac{\sin x}{x} + \frac{x^6}{x^4+1}$$

(c) Find the intervals of increase and decrease for the following function. Also, determine the points at which the function has a local maximum and local minimum.

$$f(x) = xe^{-6x^2}$$

(d) For the same $f(x)$ as in the previous part, find the intervals on which the function is concave up and concave down, and find the points of inflection.

3. (15 points) Sketch the graph of a function $f(x)$ with the following properties:

- f is continuous at all points of its domain, which is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.
- The x-intercept and y-intercept are both 0.
- $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x) = \lim_{x=-3^-} f(x) = +\infty$.
- $\lim_{x\to -3^+} f(x) = -\infty$.
- $\lim_{x\to -\infty} f(x) = +\infty$.
- $\lim_{x\to +\infty} f(x) = 0$.
- Decreasing on $(-\infty, -13) \cup (-9, -5) \cup (3, \infty)$.
- Increasing on $(-13, -9) \cup (-3, 3)$.
- Concave up on $(-\infty, -11) \cup (-7, -3) \cup (0, 3) \cup (3, \infty)$.
- Concave down on $(-11, -7) \cup (-3, 0)$.

4. (15 points) The sides of a cylindrical container with an open top are to be made of metal, while the (circular) base is to be made of wood. The sheet of metal used costs \$$3\pi$ per square centimeter while the wood costs \$1 per square centimeter.

If the container needs to have volume 9 cubic centimeters, what dimensions (i.e. what radius and what height) should one choose to minimize the cost of manufacturing the cylinder?

5. (18 points) Compute the following integrals by interpreting them geometrically and using areas from high school geometry. You will get no points if you use Riemann summation or the fundamental theorem of calculus.

(a)

$$\int_0^3 \sqrt{9 - x^2}\, dx.$$

(b)

$$\int_0^{10} (2x + 5)\, dx.$$

6. (12 points) The velocity of a particle moving in a straight line at time t is given in m/s by the formula

$$v(t) = t^2 - 4t - 12$$

(a) What is the displacement of the particle between $t = 0$ and $t = 9$?

(b) What is the distance travelled by the particle between $t = 0$ and $t = 9$?

Remember to include the units in your final answer. Time is measured in s.

7. (20 points) Compute the antiderivatives for the following functions.

(a)

$$\tan^2 x.$$

(b)

$$\sqrt[3]{x}(x^2 + x + 1).$$

(c)

$$2^x + x^2.$$

(d)

$$\frac{1}{\sqrt{1 - x^2}} + \cos x.$$

(e)

$$\sin^2 x + \cos^2 x.$$

8. (20 points) Consider the following integral:

$$\int_0^3 (x-1)^2 3^x \, dx.$$

Write a Riemann sum (using Σ notation) for this integral in which the partition has $n = 3$ subintervals of equal length and the sample points x_j^* are the right endpoints of the subintervals. Evaluate this sum, doing all the numerical and algebraic simplifications necessary.

9. (20 points) Compute the following. You are allowed to use the fundamental theorem of calculus.

(a)

$$\frac{d}{dx}\int_2^{1-x^2} e^{(1-t)^2}\, dt.$$

(b)

$$\int_0^4 (x^3 + x^{15})\, dx.$$

(c)

$$\int_5^{10} \frac{dx}{x}.$$

(d)

$$\frac{d}{dx}\int_2^{x} (t\log t)\, dt.$$