MATH 142 Exam 2 Review Sheet

Applications of Integration

- Area: If $f(x) \geq g(x)$ on an interval $[a, b]$, then the area of the region bounded by the two curves over that interval is

$$\int_a^b (f(x) - g(x))\, dx.$$

Of course, it may be that the curves cross at some point(s), in which case you have to split up the integral to make sure you're always subtracting the larger function minus the smaller function, and also it may be more convenient to integrate along the y-axis. In short,

$$\text{Area} = \int y_{\text{top}} - y_{\text{bottom}}\, dx \quad \text{or} \quad \text{Area} = \int x_{\text{right}} - x_{\text{left}}\, dy.$$

- Volume: The general idea is that if you can slice a solid into a bunch of two-dimensional cross sections, then the volume of the solid is given by

$$\text{Volume} = \int \text{cross-sectional area}.$$

We did specific versions of this when we did volumes of revolution, where the cross-sections are either washers or cylindrical shells.

Disk/Washer Method: Cross sections perpendicular to axis of revolution,

$$\text{Volume} = \pi \int r_{\text{outer}}^2 - r_{\text{inner}}^2 (dx \text{ or } dy)$$

Horizontal axis integrate dx, vertical axis integrate dy.

Cylindrical Shell Method: Cross sections parallel to axis of revolution,

$$\text{Volume} = \int 2\pi(\text{radius})(\text{height})(dx \text{ or } dy)$$

Horizontal axis integrate dy, vertical axis integrate dx.

- Work: If a constant force F acts on an object, then the work required to move the object a distance d is $W = Fd$. More generally, if a force $f(x)$ acts on the object, then the work required to move the object from $x = a$ to $x = b$ is

$$W = \int_a^b f(x)\, dx.$$

Traditional units for work are the foot-pound (pretty self-explanatory), and the Joule (J), which is a Newton-meter, and recall that a Newton is a kilogram times acceleration (in many contexts due to gravity) in m/s^2.

Hooke's Law for Springs: Hooke?s Law says that the force acting on the end of a spring stretched x units past its natural length is proportional to x, in other words $f(x) = kx$. Often for these problems, you use given information to solve for k, then use your knowledge of k to find a new piece of information.

Total Work Against Gravity: We did several examples of computing the work required to pull up a heavy cable or to pump liquid out a tank. These problems are different from the simple work formula given above because we're not moving a single object over a single distance, but rather we chop up what we want to move into a bunch of cross-sections, each of which are moved a different distance, and we use the formula

$$\text{Total Work} = \int (\text{distance traveled by slice})(\text{weight of slice}).$$

For the "weight of the slice", in cable problems we multiply the density of the cable (in lb/ft, kg/m, etc.) times dx (or whatever you called your variable), whereas for pumping problems we multiply the density of the liquid (in lb/ft^3, kg/m^3, etc) times the CROSS-SECTIONAL AREA times dx, and if the given units are pure mass (namely kg), then you also need to multiply by the gravitational constant g, whereas if the given unit is already force (namely pounds), you don't.

NOTE: For these applications, these integrals are of course all DEFINITE integrals, I just didn't put limits of integration in some of these formulas because that would've just meant more letters. The answers to all these application problems above should be NUMBERS, in fact positive numbers.

- Average Value: The average value of a function $f(x)$ over an interval $[a, b]$ is given by

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Techniques of Integration

- Integration By Parts: Differentiate one function, integrate the other, arrive at an easier integral. Formula:

$$\int u\,dv = uv - \int v\,du.$$

ALSO REMEMBER: You might have to apply the formula more than once.