MTH 165: Linear Algebra with Differential Equations

Final Exam

May 7, 2012

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the box:

Dan-Andrei Geba	MWF 10:00 - 10:50 AM	
Ang Wei	MW 2:00 - 3:15 PM	

- **The presence of electronic devices (including calculators), books, or formula cards/sheets at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Clearly circle or label your simplified final answers.**
- **You are responsible for checking that this exam has all 11 pages.**

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
TOTAL	100	

1. (10 points) Solve the following initial value problems:

(a) (5 points)

$$(e^t + 1)\,\frac{dy}{dt} + e^t\,y = 1, \quad y(0) = 1;$$

(b) (5 points)

$$y' = 1 + x + y + xy, \quad y(0) = 0.$$

2. (10 points) Consider an RC circuit which has $R = 4\,\Omega$, $C = 1/8\,F$, and $E(t) = 12\sin 3t\,V$. If the capacitor is uncharged initially, determine the current in the circuit for $t \geq 0$.

3. (10 points)

(a) (5 points) Construct two matrices A and B of the same dimensions, for which

$$\operatorname{rank}(A+B) \neq \operatorname{rank}(A) + \operatorname{rank}(B).$$

(b) (5 points) Construct two matrices A and B of appropriate dimensions, such that

$$\operatorname{rank}(AB) \neq \operatorname{rank}(A) \cdot \operatorname{rank}(B).$$

4. (10 points) For each of the following subsets of $M_{3\times 3}(\mathbb{R})$, determine whether it is a subspace. If that is the case, find its dimension.

(a) (5 points) S is the set of matrices $A \in M_{3\times 3}(\mathbb{R})$ satisfying

$$A^T = 2\,A.$$

(b) (5 points) S is the set of matrices $B \in M_{3\times 3}(\mathbb{R})$ verifying

$$B^T + 3B = 5\,I_{3\times 3}.$$

5. (10 points) Show that the vectors $v_1 = (2, -3, 5)$, $v_2 = (8, -12, 20)$, $v_3 = (1, 0, -2)$, $v_4 = (0, 2, -1)$, and $v_5 = (7, 2, 0)$ span $\mathbb{R}^3$. Find a subset of the set $\{v_1, v_2, v_3, v_4, v_5\}$ which is a basis for $\mathbb{R}^3$.

6. (10 points) Let P_2 denote the vector space of polynomials with real coefficients and degree at most 2. Define $T : P_2 \to P_2$ by

$$T(f) = f(0)\,x + f'(2)\,x^2.$$

(a) (3 points) Show that T is a linear transformation.

(b) (3 points) Determine a basis for $\mathrm{Ker}(T)$. What is $\dim[\mathrm{Ker}(T)]$?

(c) (2 points) Find $\dim[\mathrm{Rng}(T)]$.

(d) (2 points) Is the polynomial $5x + 3$ in the range of T? Explain.

7. (10 points) Consider the matrix

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}.$$

(a) (4 points) Determine its eigenvalues and their multiplicities.

(b) (4 points) Compute the eigenspaces corresponding to each of the eigenvalues and their dimensions.

(c) (2 points) Conclude, with explanation, whether A is a defective or non-defective matrix.

8. (10 points) Solve the initial value problem

$$y'' - 4y' + 8 = 0, \quad y(0) = 3, \quad y'(0) = 10.$$

9. (10 points) Determine the general solution to

$$y'' - 5y' + 6y = 4xe^{2x}.$$

10. (10 points)

(a) (5 points) Determine the annihilator for the function

$$F(x) = x\,e^{-x} + 2\cos x.$$

(b) (5 points) Using the information obtained previously, find the general solution to

$$y''' - y' = x\,e^{-x} + 2\cos x.$$