MTH 165: Linear Algebra with Differential Equations

2nd Midterm

April 5, 2012

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the box:

Dan-Andrei Geba	MWF 10:00 - 10:50 AM	
Ang Wei	MW 2:00 - 3:15 PM	

- **The presence of of electronic devices (including calculators), books, or formula cards/sheets at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Clearly circle or label your simplified final answers.**
- **You are responsible for checking that this exam has all 7 pages.**

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	60	

1. (10 points) Find the determinants of the matrices A, B, and $B^T A$, where

$$A = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 2 & 2 & 1 \\ -2 & 0 & 4 & 1 \\ 0 & -2 & 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 5 & 6 & -14 \\ 0 & 2 & 13 & -156 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

2. (10 points) In each of the following, determine whether the subset S is a subspace of the given vector space V:

i) $V = M_{2\times 2}(\mathbb{R})$ and S is the subset of all 2×2 invertible matrices;

ii) $V = P_2$, the vector space of real-valued polynomials of degree ≤ 2, and

$$S = \{ax^2 + bx \,:\, a, b \in \mathbb{R}\}.$$

3. (10 points) Compute

$$\operatorname{span}\{(1,0,-1),(2,0,4),(-5,0,2),(0,0,1)\}$$

in the vector space $\mathbb{R}^3$.

4. (10 points) Using the Wronskian, determine whether or not the functions

$$f_1(x) = e^{2x}\,,\ f_2(x) = e^{3x}\,,\ f_3(x) = e^{-x}$$

are linearly independent on $\mathbb{R}$.

5. (10 points) Let S be the subspace of $\mathbb{R}^3$ that consists of all (x, y, z) which satisfy the equation $x + 3y - 2z = 0$. Determine a basis for S and find $\dim[S]$.

6. (10 points) For the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 & 5 \\ 0 & 2 & -1 & 7 \\ 4 & 2 & -3 & 13 \end{bmatrix},$$

find:

i) a basis and the dimension for colspace$\,(A)$;

ii) a basis and the dimension for nullspace$\,(A)$.