MTH 165: Linear Algebra with Differential Equations

1st Midterm

February 21, 2013

NAME (please print legibly): ——————————————————————

Your University ID Number: ——————————————————————

Indicate your instructor with a check in the box:

Dan-Andrei Geba	MWF 10:00 - 10:50	
Giorgis Petridis	MWF 13:00 - 13:50	
Eyvindur Ari Palsson	MW 14:00 - 15:15	

- The presence of of electronic devices (including calculators), books, or formula cards/sheets at this exam is strictly forbidden.

- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- Clearly circle or label your <u>simplified</u> final answers.

- You are responsible for checking that this exam has all **7** pages.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	60	

1. (10 points) Find the explicit solution for the initial value problem

$$\frac{dy}{dx} = x^2 + x^2 y^2, \quad y(0) = 1.$$

2. (10 points) Solve the initial value problem

$$\frac{dy}{dt} + \frac{y}{2} - \frac{e^{t/3}}{2} = 0, \quad y(0) = \frac{6}{5}.$$

3. (10 points) Consider the RC circuit which has

$$R = 2\,\Omega, \qquad C = \frac{1}{8}\,F, \quad \text{and} \quad E(t) = 5\,V.$$

If $q(0) = 7$ coulombs, determine the current in the circuit for $t \geq 0$.

4. (10 points) A 200-gal tank initially contains 100 gal of pure water. Brine enters the tank through two faucets: one containing 0.2 lb/gal of salt flows in at the rate of 1 gal/min, while the second one containing 0.1 lb/gal of salt flows in at the rate of 3 gal/min. The well-stirred mixture flows out of the tank at the rate of 2 gal/min. How much salt is in the tank just before the solution overflows?

5. (10 points) Find the rank for the matrix

$$A = \begin{bmatrix} 5 & 2 & -5 \\ 9 & 4 & -7 \\ 4 & 1 & -7 \end{bmatrix}$$

by computing its **reduced row-echelon form**.

6. (10 points) Solve the following linear system of equations:

$$\begin{cases} 3x_1 + x_2 + x_3 + 6x_4 = 14 \\ x_1 - 2x_2 + 5x_3 - 5x_4 = -7 \\ 4x_1 + x_2 + 2x_3 + 7x_4 = 17 \end{cases}$$