

MTH 165: Linear Algebra with Differential Equations

First Midterm

February 25, 2014

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Freidmann	MW 16:50 - 18:05	
Karapetyan	MW 14:00-15:15	
Petridis	MWF 10:00 - 10:50	

- You have 75 minutes to work on this exam.
- No calculators, cell phones, other electronic devices, books, or notes are allowed during this exam.
- Show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- In your answers, you do not need to simplify arithmetic expressions like $\sqrt{\sin(5)}$. However, you must simplify logarithmic expressions like $\ln e$, trigonometric expressions like $\sin \pi$, and expressions like $-\frac{-1}{1/x-1}$ as much as possible.
- You are responsible for checking that this exam has all 7 pages.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	60	

By taking this exam, you are acknowledging that the following is prohibited by the College's Honesty Policy: Obtaining an examination prior to its administration. Using unauthorized aid during an examination or having such aid visible to you during an examination. Knowingly assisting someone else during an examination or not keeping your work adequately protected from copying by another.

1. (10 points) Find the general solution of the differential equation on the interval $(0, 2)$

$$x^3 \frac{dy}{dx} + 3x^2 y + \frac{2}{4-x^2} = 0.$$

Give your answer in explicit form.

2. (10 points) Solve the initial value problem on the interval $[e^{-1}, \infty)$.

$$x \ln(x) \sin(y) \frac{dy}{dx} - \sin\left(\frac{\pi}{2} - y\right) = 0, \quad y(e^{-1}) = -\pi.$$

Give your solution in implicit form.

3. (10 points) A large tank initially contains 1 L of solution in which there is dissolved 10 g of salt. A solution containing te^{-t} g/L of salt, where t is the time in minutes from the initial state, flows into the tank at a rate of 1 L/min. Through another faucet pure water flows into the tank at a rate of 1 L/min. The well-stirred mixture flows out of the tank at a rate of 2 L/min. Find $c(t)$, the concentration of salt in the well-stirred mixture after t minutes. What value does $c(t)$ tend to, when $t \rightarrow \infty$?

4. (10 points) Solve the following system of linear equations or show it is inconsistent.

$$\begin{cases} 3x_1 - 2x_2 + 12x_3 + 20x_4 = -1 \\ 2x_1 - x_2 + 8x_3 + 14x_4 = 1 \\ 2x_1 + 8x_3 + 9x_4 = -1 \end{cases} .$$

5. (10 points) Solve the following system of linear equations or show it is inconsistent.

$$\begin{cases} 3x_1 + 2x_2 - x_3 + 5x_4 = 6 \\ 4x_1 + 5x_2 + x_3 + 3x_4 = 2 \\ x_1 - 4x_2 - 5x_3 + 9x_4 = -4 \end{cases} .$$

6. (10 points) Let A be the 3×3 matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Find each of A^{-1} and $A^{-1}A^T$

or show that they do not exist. Use your answer to solve the following linear system of

equations:
$$\begin{cases} x_1 + x_3 = 1 \\ x_2 = 1 \\ x_1 + x_2 + 2x_3 = 1 \end{cases}$$