MTH 165: Linear Algebra with Differential Equations

First Midterm

February 26, 2015

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Dummit	TR 16:50-18:05	
Friedmann	MW 16:50-18:05	
Petridis	MWF 10:25-11:15	
Rice	MW 14:00-15:15	

- You have 75 minutes to work on this exam.

- No calculators, cell phones, other electronic devices, books, or notes are allowed during this exam.

- Show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- You are responsible for checking that this exam has all 7 pages.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
TOTAL	50	

1. (10 points) Solve the following initial value problem on the interval $[0, \infty)$

$$2y(1 + x^3)y' - x^2y^2 = ax^2, \ y(0) = a,$$

where $a > 0$ is a positive constant. Give your answer in implicit form.

2. (10 points) Find the general solution to

$$(e^{kx} - \frac{y}{x})dx - dy = 0,$$

on the interval $(0, \infty)$ where $k \neq 0$ is a non-zero constant. Give your answer in explicit form.

3. (10 points) A tank initially contains 10 L of a salt solution. Pure water flows into the tank at a rate of 2 L/min, and the well-stirred mixture flows out at a rate of 3 L/min. After 5 min, the concentration of salt in the tank is 25 g/L. Find:

(a) The amount of salt in the tank initially.

(b) The volume of solution in the tank when the concentration of salt is 4 g/L.

4. (10 points) Determine all values of the constant a for which the following system has
i) no solution, **ii)** an infinite number of solutions, **iii)** a unique solution.

$$x_1 + x_2 - x_3 = 3$$
$$2x_1 + 5x_2 - 4x_3 = 10$$
$$2x_1 + 3x_2 + ax_3 = 0$$

5. (10 points)

(a) Determine the inverse of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}.$$

(b) Let

$$B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 0 & -2 \\ -1 & 3 & 2 \end{bmatrix}.$$

Find the 3×3 matrix X that satisfies the matrix equation $XA + B = 0$, where 0 is the 3×3 zero matrix.

(c) What is the rank of the matrix A? Justify your answer briefly.