MTH 165: Linear Algebra with Differential Equations

2nd Midterm

April 2, 2015

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the box:

Dummit	TR 16:50-18:05	
Friedmann	MW 16:50-18:05	
Petridis	MWF 10:25-11:15	
Rice	MW 14:00-15:15	

- You have 75 minutes to work on this exam.
- No calculators, cell phones, other electronic devices, books, or notes are allowed during this exam.
- Show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You are responsible for checking that this exam has all 6 pages.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
TOTAL	50	

1. (10 points)

(a) Let

$$A = \begin{bmatrix} -1 & 3 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}.$$

Find the determinant of the matrix $C = ABA^2B^T$.

(b) Use the Wronskian to determine whether the functions $f_1(x) = \cos(x)$, $f_2(x) = \sin(x)$, and $f_3(x) = x$ are linearly independent.

2. (10 points) Determine whether each given set S is a subspace of the given vector space V. If so, give a proof; if not, explain why not.

(a) $V = \mathbb{R}^3$, and $S = \{(x, y, z) \in V \mid \ x^2 + y^2 + z^2 = 1\}$.

(b) $V = M_2(\mathbb{R})$, the set of 2×2 matrices, and $S = \{A \in V \mid \ \det(A) = 0\}$.

(c) $V = P_2(\mathbb{R})$, the set of polynomials of degree ≤ 2, and $S = \{f \in V \mid f(2) = 2f(1)\}$.

3. (10 points) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, and $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

(a) Do these vectors span $\mathbb{R}^3$? Explain why or why not.

(b) Are these vectors linearly independent? If so, justify why; if not, find an explicit linear dependence between them.

4. (10 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 1 & 3 \\ 2 & 4 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

5. (10 points) Answer the following about a 6×17 matrix A (that is, a matrix with 6 rows and 17 columns) such that $\text{rank}(A) = 6$.

(a) rowspace(A) is a ______-dimensional subspace of $\mathbb{R}^d$ with $d =$______

(b) colspace(A) is a ______-dimensional subspace of $\mathbb{R}^d$ with $d =$______

(c) nullspace(A) is a ______-dimensional subspace of $\mathbb{R}^d$ with $d =$______

(d) Are the rows of A linearly independent? Explain why or why not.

(e) Are the columns of A linearly independent? Explain why or why not.