MATH 165 (SUMMER '22, SESH B2)

REMAINING OFFICE HOURS :

PABLO :

W – 9:30 PM - 10:30 PM (ET)

LECTURES :

9:00 AM – 11:15 AM (ET)

M, T, W, R

Zoom ID :

979-4693-0650

NOTE : ALL IMAGES ARE FROM (GOODE & ANNIN 4TH EDITION)

COURSE WEB PAGE

https://people.math.rochester.edu/grads/asahay/summer2022/math165/index.html

SHORT URL : bit.ly/sahay165

ANNOUNCEMENTS / NOTES

1. AFFIS SURVEY IS AVAILABLE. → INSTRUCTOR → COURSE

2. WW 10 - IS DUE WED (3rd AUG) AT 11:00 PM ET
   WW 11 - IS DUE FRI (5th AUG) AT 11:00 PM ET

3. HARD WEBWORK DEADLINE : FRIDAY, 5th AUG

4. WW 11 IS EXTRA CREDIT. A - SCORE WITHOUT WW11, S = MAX WITHOUT WW11
   B - SCORE IN WW 11

$$\mathrm{MAX}\left(\frac{A+B}{S}, 1\right) * 25$$

5. EXTRA OFFICE HOURS (SEE PREV. PAGE.)

ANNOUNCEMENTS / NOTES

MAY CHANGE

6. FINAL EXAM ON THURSDAY (SCHEDULER + SAMPLE)

7. REMINDER : PLEASE KEEP VIDEOS ON, IF POSSIBLE!

5.(a) $V = P_{10}(\mathbb{R})$

$$S = \{ \underset{f_1}{\underset{\downarrow}{1}},\ \underset{f_2}{\underset{\downarrow}{x+1}},\ \underset{f_3}{\underset{\downarrow}{(x+2)^2}} \}$$

$$W[f_1, f_2, f_3](x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_1'(x) & f_2'(x) & f_3'(x) \\ f_1''(x) & f_2''(x) & f_3''(x) \end{vmatrix} = \begin{vmatrix} 1 & x+1 & (x+2)^2 \\ 0 & 1 & 2(x+2) \\ 0 & 0 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x+1 & (x+2)^2 \\ 0 & 1 & 2(x+2) \\ 0 & 0 & 2 \end{vmatrix} = 1 \cdot 1 \cdot 2 = 2 \neq 0$$

UPPER TRIANGULAR.

$1, x+1, (x+2)^2 \longrightarrow$ L.I.

5.(b) $V = C^2(-\infty, \infty)$

$$S = \{ \underset{f_1}{1}, \underset{f_2}{\cos^2 x}, \underset{f_3}{\cos 2x} \}$$

$$W[f_1, f_2, f_3](x) = \begin{vmatrix} 1 & \cos^2 x & \cos 2x \\ 0 & -2\cos x \sin x & -2\sin 2x \\ 0 & 2(\sin^2 x - \cos^2 x) & -4\cos 2x \end{vmatrix}$$

$$\frac{d}{dx}(-2\cos x \sin x)$$

$$= 2(\sin x)^2 - 2(\cos^2 x)$$

$$\begin{vmatrix} 1 & \cos^2 x & \cos 2x \\ 0 & -2\cos x \sin x & -2\sin 2x \\ 0 & 2(\sin^2 x - \cos^2 x) & -4\cos 2x \end{vmatrix} = \begin{vmatrix} -2\cos x \sin x & -2\sin 2x \\ 2(\sin^2 x - \cos^2 x) & -4\cos 2x \end{vmatrix}$$

$$= 8\cos x \sin x \cos 2x - 4\sin^2 x \sin 2x + 4\sin 2x \cos^2 x$$

$$= 0$$

$$1,\ \cos^2 x,\ \cos 2x, \qquad \cos 2x = 2\cos^2 x - 1$$

$$\boxed{\cos 2x - 2\cos^2 x + 1 = 0}$$

L.D.

3(c)

$V = V_2(0,1)$

$S = \{\vec{x}_1, \vec{x}_2\}$

$\vec{x}_1(t) = \begin{bmatrix} t \\ t^2 \end{bmatrix}$ $\vec{x}_2(t) = \begin{bmatrix} t^2 \\ t \end{bmatrix}$

$W[\vec{x}_1, \vec{x}_2](t) = \det \begin{bmatrix} \vec{x}_1 & \vec{x}_2 \end{bmatrix} = \begin{vmatrix} t & t^2 \\ t^2 & t \end{vmatrix} = t^2 - t^4$

$t = 2$

$\vec{x}_1, \vec{x}_2 \longrightarrow$ L. I.

$2^2 - 2^4 = -12 \neq 0$

6. $\lambda = 2$

$$A = \begin{bmatrix} 0 & 2 & -2 \\ -2 & 4 & -2 \\ -2 & 2 & 0 \end{bmatrix} \rightarrow A - \lambda I = \begin{bmatrix} -2 & 2 & -2 \\ -2 & 2 & -2 \\ -2 & 2 & -2 \end{bmatrix}$$

RANK

$$(A - \lambda I)\vec{v} = 0 \rightarrow -2v_1 + 2v_2 - 2v_3 = 0$$

RANK = 1

OF VAR = 3

DEG OF FREEDOM = 2

$$v_1 = \frac{2v_2 - 2v_3}{2} = v_2 - v_3 = s - t$$

$v_2 = s$

$v_3 = t$

$$v = (s-t,\ s,\ t) \qquad (s,t \in \mathbb{R})$$

$$= (s,s,0) + (-t,0,t)$$

$$= s(1,1,0) + t(-1,0,1)$$

$$\text{EIGENSPACE}(A) = \text{SPAN}\{\underbrace{(1,1,0),\ (-1,0,1)}_{\text{L.I. e.v. OF } \lambda = 2}\}$$

$\vec{v}_1$ & $\vec{v}_2$

$$\left.\begin{aligned} A\vec{v}_1 &= \lambda\vec{v}_1 \\ A\vec{v}_2 &= \lambda\vec{v}_2 \end{aligned}\right) \Rightarrow A(\vec{v}_1+\vec{v}_2) = A\vec{v}_1 + A\vec{v}_2$$

$$= \lambda\vec{v}_1 + \lambda\vec{v}_2 = \lambda(\vec{v}_1+\vec{v}_2)$$

7. $P(D) = D^2 + 3D + 2$

$Q(D) = D + 1$

(b) SOLVE $P(D)Q(D)\,y = 0$

$$(D^2 + 3D + 2)(D+1)\,y = 0$$

AUXILIARY POLYNOMIAL $= (r^2 + 3r + 2)(r+1)$

$(r+1)(r+2)(r+1) = (r+1)^2(r+2)$

$$P(D)\,Q(D)\,y = 0$$

$$\underbrace{\lambda=-1, \quad \lambda=-1, \quad \lambda=-2 \qquad e^{-x} \quad x e^{-x} \quad e^{-2x}}_{\text{L.I.}}$$

$$(D-\lambda)^m \longrightarrow e^{\lambda x},\ x e^{\lambda x},\ \dots,\ x^{m-1} e^{\lambda x}$$

GEN. SOLN $\Rightarrow$ $y = c_1 e^{-x} + c_2 x e^{-x} + c_3 e^{-2x}$

(c) USING PREV. ANSWER FIND GEN. SOLN. TO

$$P(D)\,y = e^{-x}$$

$$(D^2 + 3D + 2)\,y = e^{-x}$$

MULTIPLY BY $Q(D)$

$$\Rightarrow (D^2+3D+2)(D+1)\,y = (D+1)\,e^{-x} = 0$$

$$y = c_1 e^{-x} + \boxed{c_2 x e^{-x}} + c_3 e^{-2x}$$

y_p

y_c

NOT A PARAMETER

$$P(D)y = 0 \longrightarrow (D+2)(D+1)y$$

$$y_c = c_1 e^{-x} + c_3 e^{-2x}$$

PLUG $y_p = c_2 x e^{-x}$ INTO $P(D)y = e^{-x}$

$$Dy_p = \frac{d}{dx}\left(c_2 x e^{-x}\right) = c_2 e^{-x} - c_2 x e^{-x}$$

$$D^2 y_p = \frac{d}{dx}\left(c_2 e^{-x} - c_2 x e^{-x}\right) = -2c_2 e^{-x} + c_2 x e^{-x}$$

LHS $\quad P(D)\,y_p = (D^2 + 3D + 2)\,y_p = D^2 y_p + 3D y_p + 2 y_p$

$$= -2c_2 e^{-x} + c_2 x e^{-x} + 3(c_2 e^{-x} - c_2 x e^{-x})$$

$$+ 2(c_2 x e^{-x})$$

$$= c_2 e^{-x}(-2 + 3) + \cancel{c_2 x e^{-x}(1 - 3 + 2)}$$

$$= c_2 e^{-x}$$

$$\stackrel{?}{=}$$

RHS $= e^{-x}$

$$\Rightarrow \quad c_2 = 1$$

$y = y_p + y_c$

$$\boxed{y = xe^{-x} + c_1 e^{-x} + c_3 e^{-2x}}$$

GEN. SOLN.

BREAK TILL

10:05 AM

$$(D^2+3D+2)(D+1)\,y = 0$$

$$D^3+3D^2+2D+D^2+3D+2$$

$$=(D^3+4D^2+5D+2)\,y = 0$$

$$\frac{dx_3}{dt} \leftarrow \boxed{\frac{d^3y}{dt^3}} + 4\underbrace{\boxed{\frac{d^2y}{dt^2}}}_{\to x_3} + 5\underbrace{\boxed{\frac{dy}{dt}}}_{\to x_2} + 2\underbrace{\boxed{y}}_{\to x_1} = 0$$

$$x_1 = y \qquad x_2 = \frac{dy}{dt} \qquad x_3 = \frac{d^2y}{dt^2}$$

$$\left.\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= x_3 \\ \frac{dx_3}{dt} &= -4x_3 - 5x_2 - 2x_1 \end{aligned}\right\} \text{SYSTEM.}$$

$$\frac{d\vec{x}}{dt} = \overbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix}}^{A} \vec{x}$$

E.V. & E.V. $(\lambda, \vec{v}) \to$ EIGENPAIR FOR A

$\vec{x} = e^{\lambda t}\vec{v}$ IS A SOLN.

FOR $\frac{d\vec{x}}{dt} = A\vec{x}$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ -2 & -5 & -4-\lambda \end{vmatrix}$$

$$= -\lambda \begin{vmatrix} -\lambda & 1 \\ -5 & -4-\lambda \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ -2 & -4-\lambda \end{vmatrix}$$

$$= -\lambda(\lambda^2 + 4\lambda + 5) - 2 = -\lambda^3 - 4\lambda^2 - 5\lambda - 2$$

$$-\lambda^3 - 4\lambda^2 - 5\lambda - 2 = -(\lambda+1)(\lambda^2 + 3\lambda + 2)$$

$$= -\underbrace{(\lambda+1)^2}_{\text{NO REPEATS IN ACTUAL EXAM.}}\underbrace{(\lambda+2)}_{\lambda = -2}$$

$$-(-1)^3 - 4(-1)^2 - 5(-1) - 2$$

$$= 1 - 4 + 5 - 2 = 0$$

$$\lambda = -2$$

$$A - \lambda I = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ -2 & -5 & -2 \end{vmatrix}$$

$$(A - \lambda I)\vec{v} = 0$$

$$\xrightarrow{\text{ERO's}} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$2v_1 + v_2 = 0$$

$$2v_2 + v_3 = 0$$

$$v_1 = t$$

$$v_2 = -2t$$

$$v_3 = 4t$$

$$\vec{v} = (t, -2t, 4t) = t\boxed{(1, -2, 4)}$$

$$\vec{x} = e^{-\lambda t}\vec{v}$$

$$(x_1, x_2, x_3) = e^{-2t}(1, 2, -4)$$

$$y = x_1$$

$$\boxed{y = e^{-2t}}$$

7. $A = \begin{bmatrix} 7 & -2 \\ 2 & 7 \end{bmatrix}$

$\boxed{\lambda = 7+2i}$ $\qquad \lambda = 7-2i$

$(A - \lambda I)\vec{v} = 0$

$$A - \lambda I = \begin{bmatrix} 7-(7+2i) & -2 \\ 2 & 7-(7+2i) \end{bmatrix} = \begin{bmatrix} -2i & -2 \\ 2 & -2i \end{bmatrix}$$

$$\begin{bmatrix} -2i & -2 \\ 2 & -2i \end{bmatrix} \xrightarrow{A_{12}(-i)} \begin{bmatrix} -2i & -2 \\ 0 & 0 \end{bmatrix}$$

$$-2i\,v_1 \quad -2v_2 = 0$$

$$v_1 = t \qquad \Rightarrow \quad v_2 = \frac{-2iv_1}{2} = -it$$

$$\therefore \quad \vec{v} = (t, -it) = t(1, -i)$$

$$(7+2i) \longleftrightarrow (1, -i)$$

$$\lambda = 7-2i \quad , \quad \vec{v} = (1, i)$$

$$\vec{x} = e^{\lambda t}\vec{v}$$

$$\vec{x}_1 = e^{(7+2i)t}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$\vec{x}_2 = e^{(7-2i)t}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2$$

$$\vec{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$c_1 \vec{x}_1(0) + c_2 \vec{x}_2(0) = \vec{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$c_1 \left(\underbrace{\cancel{e^{(7+2i)\cdot 0}}}_{1} \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix} \right) + c_2 \left(\underbrace{\cancel{e^{(7-2i)\cdot 0}}}_{1} \begin{bmatrix} 1 \\ i \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 \\ -ic_1 \end{bmatrix} + \begin{bmatrix} c_2 \\ ic_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$c_1 + c_2 = 0 \quad , \quad -ic_1 + ic_2 = 1 \Rightarrow 2ic_2 = 1$$

$$\Rightarrow c_2 = \frac{1}{2i} = -\frac{i}{2}$$

$$c_1 = -c_2 = i/2$$

$$c_1 = i/2 \qquad\qquad c_2 = -i/2$$

$$\vec{x} = c_1 x_1 + c_2 x_2$$

$$= \frac{i}{2} e^{(7+2i)t} \begin{bmatrix} 1 \\ -i \end{bmatrix} - \frac{i}{2} e^{7-2it} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$= \frac{e^{7t}}{2} \left((\cos 2t + i \sin 2t) \begin{bmatrix} i \\ 1 \end{bmatrix} + (\cos 2t - i \sin 2t) \begin{bmatrix} -i \\ 1 \end{bmatrix} \right)$$

$$\frac{e^{7t}}{2}\left(\left(\cos 2t + i\sin 2t\right)\begin{bmatrix} i \\ 1 \end{bmatrix} + \left(\cos 2t - i\sin 2t\right)\begin{bmatrix} -i \\ 1 \end{bmatrix}\right)$$

CANCEL

$$= \frac{e^{7t}}{2}\left(\begin{bmatrix} -\sin 2t + i\cos 2t \\ \cos 2t + i\sin 2t \end{bmatrix} + \begin{bmatrix} -\sin 2t - i\cos 2t \\ \cos 2t - i\sin 2t \end{bmatrix}\right)$$

CANCEL

$$= \frac{e^{7t}}{2}\begin{bmatrix} -2\sin 2t \\ 2\cos 2t \end{bmatrix} = \begin{bmatrix} -e^{7t}\sin 2t \\ e^{7t}\cos 2t \end{bmatrix}$$

$$\boxed{\vec{x}(t) = \begin{bmatrix} -e^{7t}\sin 2t \\ e^{7t}\cos 2t \end{bmatrix}}$$

$$e^{(7+2i)t} = e^{7t+2it}$$

$$= e^{7t} \cdot e^{2it}$$

$$= e^{7t}\left[\cos(2t) + i\sin(2t)\right]$$

$$e^{ix} = \cos x + i\sin x$$

3. $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix}$

$$[\,A \mid I\,] \xrightarrow{\text{EROs}} [\,I \mid A^{-1}\,]$$

$$\left[\begin{array}{ccc|ccc} \boxed{2} & 3 & 0 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} \boxed{2} & 3 & 0 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\Big\downarrow P_{12}$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 2 & 3 & 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\Big\downarrow \; A_{12}(-2),\ A_{13}(-2)$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & \boxed{7} & 2 & 1 & -2 & 0 \\ 0 & 4 & 1 & 0 & -2 & 1 \end{array}\right]$$

$$\Big\downarrow \; A_{32}(-2)$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & \boxed{-1} & 0 & 1 & 2 & -2 \\ 0 & 4 & 1 & 0 & -2 & 1 \end{array}\right]$$

$$\Big\downarrow \; M_2(-1)$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & \boxed{1} & 0 & -1 & -2 & 2 \\ 0 & 4 & 1 & 0 & -2 & 1 \end{array}\right]$$

$$\Big\downarrow \; A_{23}(-4) \quad A_{21}(2)$$

$$A_{31}(1) \Big\downarrow \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & -3 & 4 \\ 0 & 1 & 0 & -1 & -2 & 2 \\ 0 & 0 & \boxed{1} & 4 & 6 & -7 \end{array}\right]$$

$$\downarrow A_{31}(1)$$

$$\left[\begin{array}{ccc|c} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \boxed{1} \end{matrix} & \underbrace{\begin{matrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 4 & 6 & -7 \end{matrix}}_{A^{-1}} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 4 & 6 & -7 \end{bmatrix}$$

2.(a) $\frac{dy}{dx} + 3x^2 y = 3x^5$

$$\frac{d}{dx}\left(e^{x^3} y\right) = 3x^5 e^{x^3}$$

$$e^{x^3} y = \int 3x^5 e^{x^3} dx = \int \underbrace{x^3 e^{x^3}}_{ue^u} \underbrace{(3x^2 dx)}_{du}$$

$u = x^3 \qquad du = 3x^2 dx$

$$\int u e^u du$$