MATH 165 (SUMMER '22, SESH B2)

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OFF HRS:
M – 9:00 PM – 10:00 PM (ET)
F – 3:00 PM – 4:00 PM (ET)

LECTURES:
9:00 AM – 11:15 AM (ET)
M, T, W, R

Zoom ID:
979-4693-0650

COURSE WEB PAGE $\boxed{\text{https://people.math.rochester.edu/grads/asahay/summer2022/math165/index.html}}$

SHORT URL: bit.ly/sahay165

NOTE: ALL IMAGES ARE FROM THE (GOODE & ANNIN 4TH EDITION)

ANNOUNCEMENTS / NOTES

1. MATERIALS FOR LECTURE 3 ARE UPLOADED.
2. WW 01 – DUE SATURDAY (2nd JULY) AT 11:00PM ET
   WW 02 – DUE TUESDAY (5th JULY) AT 11:00 PM ET
3. OFFICE HOURS START TOMORROW
4. $\boxed{\text{EXAM TIMES}}$ TO BE REVISED : KEEP AN EYE OUT FOR A SURVEY!
5. REMINDER : PLEASE KEEP VIDEOS ON, IF POSSIBLE!

§ 1.6 FIRST-ORDER LINEAR DIFF. EQNS.

DEFINITION 1.6.1

A differential equation that can be written in the form

$$a(x)\frac{dy}{dx} + b(x)y = r(x), \tag{1.6.1}$$

where $a(x)$, $b(x)$, and $r(x)$ are functions defined on an interval (a, b), is called a **first-order linear differential equation**.

We assume that $a(x) \neq 0$ on (a, b) and divide both sides of (1.6.1) by $a(x)$ to obtain the **standard form**

$$\frac{d}{dx}\left(I(x)y\right) \longleftarrow \frac{dy}{dx} + p(x)y = q(x), \tag{1.6.2}$$

"INTEGRATING FACTOR"

$$\boxed{\frac{dy}{dx} + p(x)\,y = q(x)} \quad \text{—} \quad \mathrm{I}$$

WANT TO MULTIPLY BY $I(x)$ SUCH THAT THE LEFT HAND SIDE BECOMES

$$\frac{d}{dx}\left(I(x)\, y \right)$$

$$\frac{d}{dx}\left(I(x)y\right) \leftarrow \boxed{I(x)\frac{dy}{dx} + I(x)p(x)\,y} = I(x)\,q(x)$$

$$\frac{dI}{dx} = p(x)\,I(x) \xrightarrow[\text{VAR"}]{\text{"SEP OF}} \boxed{I(x) = e^{\int p(x)\,dx}}$$

NOTE: DON'T MEMORIZE THE FINAL FORMULA

Example 1.6.3 Solve the initial-value problem

$$\frac{dy}{dx} + xy = xe^{x^2/2}, \quad \boxed{y(0) = 1.}$$

$p(x) = x$

$\int x \, dx = \frac{x^2}{2}$

$I(x) = e^{x^2/2}$

$$\frac{d}{dx}\left(e^{x^2/2} y\right) = e^{x^2/2}\frac{dy}{dx} + xe^{x^2/2} y = xe^{x^2}$$

$$\frac{d}{dx}\left(e^{x^2/2}\, y\right) = x e^{x^2}$$

(ANTI-DER.) $\Rightarrow$ $$e^{x^2/2}\, y = \underbrace{\int x e^{x^2}\, dx} + C = \frac{e^{x^2}}{2} + C$$

$$\int x e^{x^2}\, dx \qquad u = x^2 \qquad du = 2x\,dx$$

$$= \int \frac{1}{2}\left(e^{u}\right) du = \frac{e^{u}}{2} = \frac{e^{x^2}}{2}$$

$$y = \frac{e^{x^2}}{2e^{x^2/2}} + \frac{c}{e^{x^2/2}}$$

$$\boxed{y(x) = \frac{1}{2} e^{x^2/2} + c\, e^{-x^2/2}}$$

GENERAL SOLN.

PLUG IN $x=0$, $y(0)=1$

$$1 = y(0) = \frac{1}{2} \underbrace{e^{0^2/2}}_{1} + c\, \underbrace{e^{-0^2/2}}_{1} = \frac{1}{2} + c$$

$$\Rightarrow c = 1/2$$

SOLN
TO
IVP :

$$y(x) = \frac{1}{2}\left[e^{x^2/2} + e^{-x^2/2} \right]$$

STANDARDIZE

Example 1.6.4 Solve $x\frac{dy}{dx} - 2y = 2x^2 \ln x$, $\boxed{x > 0.}$

IMPORTANT

$p(x)$

$q(x) =$

$$\frac{dy}{dx} - \frac{2}{x} y = 2x \ln x$$

$$\int p(x)\,dx = \int \frac{-2}{x}\,dx = -2 \ln x$$

$$I(x) = e^{\int p(x)\,dx} = e^{-2\ln x}$$

$$a^{bc} = (a^b)^c$$
$$a = e$$
$$b = \ln x$$
$$c = -2$$

$$= \left(e^{\ln x}\right)^{-2} = x^{-2} = \frac{1}{x^2}$$

$$\left(\frac{dy}{dx} - \frac{2y}{x} = 2x\ln x\right) \times \left[I(x) = 1/x^2\right]$$

$$\frac{1}{x^2}\frac{dy}{dx} - \frac{2y}{x^3} = \frac{2\ln x}{x}$$

$$\left(\frac{d}{dx}\left(\frac{y}{x^2}\right) = \right) \quad \frac{1}{x^2}\frac{dy}{dx} - \frac{2y}{x^3} = \frac{2\ln x}{x}$$

$$\frac{1}{x^2}\frac{dy}{dx} + y\underbrace{\left(\frac{d\left(1/x^2\right)}{dx}\right)}_{-\frac{2}{x^3}}$$

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = \frac{2\ln x}{x}$$

$$\Rightarrow \frac{y}{x^2} = \underbrace{\int \frac{2\ln x}{x}\,dx}_{} + C$$

$$\frac{y}{x^2} = (\ln x)^2 + C$$

GENERAL SOLN

$$\Rightarrow y = x^2(\ln x)^2 + Cx^2$$

(NEXT PAGE)

$u = \ln x$

$du = \frac{dx}{x}$

$$\int \frac{2\ln x}{x}\,dx = \int 2u\,du = u^2 = (\ln x)^2$$

$$\mathcal{J} \leftarrow \int \underbrace{2\ln x}_{u}\,\underbrace{\frac{dx}{x}}_{dv} = 2(\ln x)^2 - \boxed{\int \frac{2\ln x}{x}\,dx} \;\; \mathcal{J}$$

$du = \frac{2}{x}$ $\qquad$ $v = \ln x$

$$\mathcal{J} = 2(\ln x)^2 - \mathcal{J} \Rightarrow \mathcal{J} = (\ln x)^2$$

SKIP

Example 1.6.5 Solve the initial-value problem

$$y' - y = \boxed{f(x),} \qquad y(0) = 0,$$

where $f(x) = \begin{cases} 1, & \text{if } x < 1, \\ 2 - x, & \text{if } x \geq 1. \end{cases}$

§ 1.7 MODELLING PROBLEMS USING 1st ORDER LINEAR ODEs

MIXING PROBLEM:

$$c_j \to \frac{\text{AMOUNT OF CHEMICAL (IN g)}}{\text{VOLUME OF CHEMICAL}}$$

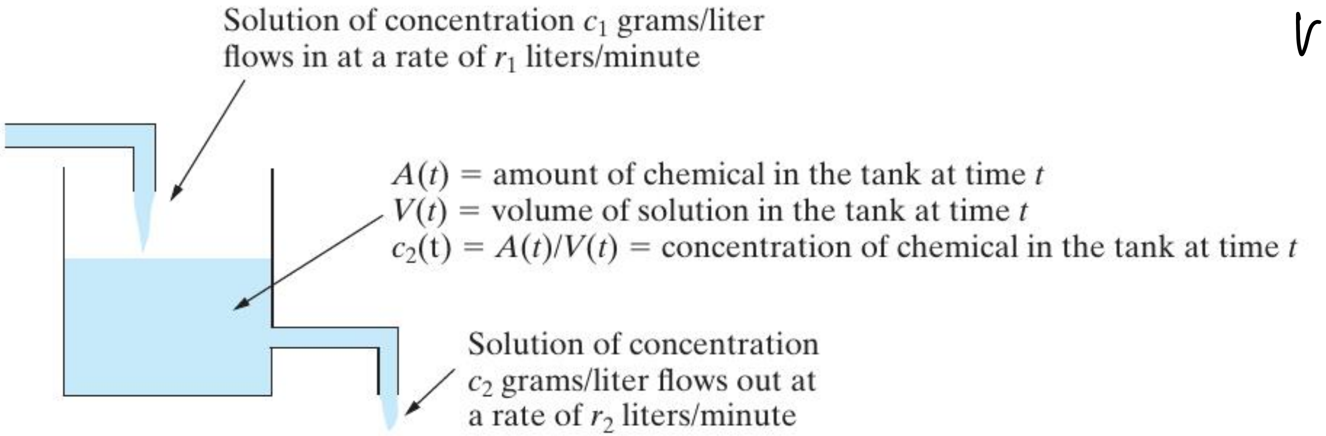


Figure 1.7.1: A mixing problem.

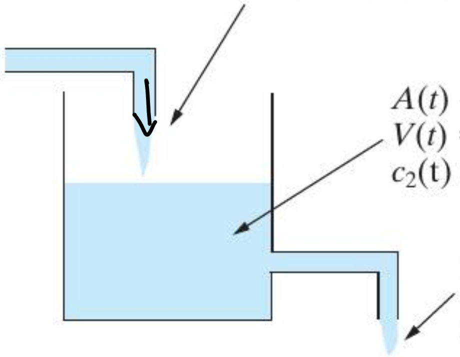


Figure 1.7.1: A mixing problem.

$$c_2 = \frac{A}{V}$$

DIFF. EQN.

$$\frac{dV}{dt} = \underbrace{r_1}_{\text{(IN-FLOW)}} - \underbrace{r_2}_{\text{(OUT-FLOW)}}$$

$$\frac{dA}{dt} = \underbrace{c_1 r_1}_{\text{(IN-FLOW)}} - \underbrace{\boxed{c_2} r_2}_{\text{(OUT-FLOW)}}$$

CONCENTRATION OF THE TANK

$$\left(c_2 = \frac{A}{V} \right)$$

$$\boxed{\frac{dV}{dt} = r_1 - r_2}$$

$$\boxed{\frac{dA}{dt} = c_1 r_1 - r_2 \frac{A(t)}{V(t)}}$$

SOLVE :

$$\int_{V_0}^{V(t)} dV = \int_0^t (r_1 - r_2)\, dt$$

$V_0 = V(0)$

$$\Rightarrow V(t) = (r_1 - r_2)\, t + V_0$$

$$\boxed{\frac{dA}{dt} = c_1\lambda_1 - \lambda_2 \frac{A}{V_0 + (\lambda_1 - \lambda_2)t}}$$

$t \rightarrow$ TIME / INDEPENDENT

$A \rightarrow$ AMOUNT AT TIME t / DEPENDENT

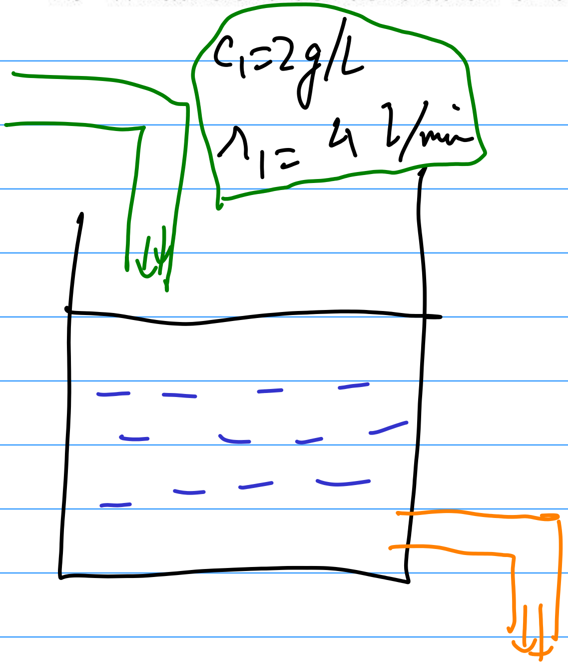
1st ORDER, LINEAR ODE.

Example 1.7.1 A tank contains 8 L (liters) of water in which is dissolved 32 g (grams) of chemical. A solution containing 2 g/L of the chemical flows into the tank at a rate of 4 L/min, and the well-stirred mixture flows out at a rate of 2 L/min.

1. Determine the amount of chemical in the tank after 20 minutes.
2. What is the concentration of chemical in the tank at that time?

$A(20)$?

$$C_2(20) = \frac{A(20)}{V(20)}$$



$$\boxed{V_0 = 8L}$$

$$\boxed{A_0 = 32g}$$

$$C_2 = A/V$$

$$\boxed{r_2 = 2\,L/min}$$

$$\frac{dV}{dt} = r_1 - r_2 = 4 - 2 = 2 \ L/_{min}$$

(INTEGRATING + I.V. DATA)

$$\int_8^{V(t)} dV = \int_0^t 2\,dt$$

$$\Rightarrow V(t) - 8 = 2(t-0) \Rightarrow \boxed{V(t) = 2t + 8}$$

B

$$\frac{dA}{dt} = c_1 r_1 - c_2 r_2 = (2)(4) - \left(\frac{A}{V}\right)(2) = 8 - \frac{2A}{2t+8}$$

$$\frac{dA}{dt} = 8 - \frac{A}{t+4}$$

STANDARDIZE

$$\text{①} - \boxed{\frac{dA}{dt} + \frac{A}{t+4} = 8}$$

y, x

$$\frac{dy}{dx} + P(x)\,y = q(x)$$

$$P(t) = \frac{1}{t+4} \Rightarrow \int p(t)\,dt = \int \frac{dt}{t+4} = \ln(t+4)$$

$$\int p(t)\,dt = \ln(t+4)$$

$$I(t) = e^{\ln(t+4)} = t+4$$

$\downarrow$ MULTIPLY (1) WITH $I(t)$

$$\underbrace{(t+4)\frac{dA}{dt} + A}_{\frac{d}{dt}\left((t+4)A\right)} = 8(t+4)$$

$$\Rightarrow \frac{d}{dt}\left((t+4)A\right) = 8(t+4)$$

$$\int_0^t \frac{d}{dt}\left[(t+4)\,\dot{A}(t)\right] dt = \int_0^t 8\,(t+4)\,dt$$

$$(t+4)\,A(t)\Big]_0^t = \left(4t^2 + 32t\right) - (0)$$

$$\overset{\shortparallel}{(t+4)\,A(t) - (4)(32)} \Rightarrow (t+4)\,A(t) - 128 = 4t^2 + 32t$$

$$\left(\because A(0) = A_0 = 32g/L\right) \qquad \Rightarrow \boxed{A(t) = \frac{4t^2 + 32t + 128}{t+4}}$$

$$\hookrightarrow \text{A}$$

PLUG IN $t = 20$ IN A & B

A : $A(20) = \dfrac{4(20^2) + 32(20) + 128}{20+4}$

$= \dfrac{1600 + 640 + 128}{24}$

$= \dfrac{2368}{24} = \dfrac{296}{3}$

B : $V(20) = 2(20) + 8 = 48$

CONC. $= \dfrac{296/3}{48}$

$= ???$

SKIP

1. A tank initially contains 600 L of solution in which there is dissolved 1500 grams of chemical. A solution containing 5 g/L of the chemical flows into the tank at a rate of 6 L/min, and the well-stirred mixture flows out at a rate of 3 L/min. Determine the concentration of chemical in the tank after one hour.

BREAK TILL

10 : 20 AM

§ 2.1 MATRICES : DEFN & NOTATION

GOAL : UNDERSTANDING LINEAR EQUATIONS

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = \underbrace{b}_{\text{CONSTANT}}$$

(CONSTANT)

$a_1, \ldots, a_n \to$ COEFFICIENTS

$x_1, \ldots, x_n \to$ VARIABLES

SYSTEM : MANY LINEAR EQUATIONS IN THE SAME VARIABLES e.g.

$$\begin{aligned} 5x_1 - x_2 - 3x_3 &= -1, \\ -x_1 + 4x_2 - 8x_3 &= 2, \\ 6x_1 \phantom{{}+4x_2} + 8x_3 &= -5. \end{aligned}$$

3 LINEAR EQUATIONS IN 3 VARIABLES.

$\Downarrow$

$x_1, x_2, x_3 \to$ VARIABLES

$$\left[\begin{array}{ccc|c} 5 & -1 & -3 & -1 \\ -1 & 4 & -8 & 2 \\ 6 & 0 & 8 & -5 \end{array}\right]$$

(AUGMENTED) MATRIX

OF ROWS

OF COLUMNS

DEFINITION 2.1.1

An $m \times n$ (read "m by n") **matrix** is a rectangular array of numbers arranged in m horizontal rows and n vertical columns. Matrices are usually denoted by upper case letters, such as A and B. The entries in the matrix are called the **elements** of the matrix.

The following are examples of a 3×3 and a 4×2 matrix, respectively:

$$A = \begin{bmatrix} 9 & 3 & -2 \\ -5 & 2 & 0 \\ 0 & -7 & 8 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 0 \\ 3 & -5 \\ -6 & 7/2 \\ -1 & -3 \end{bmatrix}.$$

□

$A \to m \times n$

$a_{ij} \to$ ith ROW
jth COLUMN

INDEX NOTATION

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & \vdots & & \ddots & \vdots \\ \vdots & \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} \end{bmatrix} = [a_{ij}] \simeq [a_{ij}]_{\substack{1 \le i \le m \\ 1 \le j \le n}}$$

$m, n \longrightarrow$ DIMENSIONS

$m \times n \longrightarrow$ SIZE

$[3 \quad 5] \to 1\times 2$ MATRIX

$\begin{bmatrix} 3 \\ 5 \end{bmatrix} \to 2\times 1$ MATRIX

EQUALITY OF MATRICES

DEFINITION 2.1.3

Two matrices A and B are **equal**, written $A = B$, if

1. They both have the same size, $m \times n$.
2. All corresponding elements in the matrices are equal: $a_{ij} = b_{ij}$ for all i and j with $1 \leq i \leq m$ and $1 \leq j \leq n$.

e.g.

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -6 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -6 & 2 \\ \boxed{0} & 3 \\ -2 & -1 \end{bmatrix}$$

2×3 3×2

$$C = \begin{bmatrix} -6 & 0 \\ \boxed{2} & -2 \\ 3 & -1 \end{bmatrix}$$

$B \neq C$

ROW & COLUMN VECTORS

> **DEFINITION 2.1.4**
>
> A $1 \times n$ matrix is called a **row n-vector**. An $n \times 1$ matrix is called a **column n-vector**. The elements of a row or column n-vector are called the **components** of the vector.

(ROW n-VECTOR) $\begin{bmatrix} - & - & - & - & - \end{bmatrix}$ / (COLUMN n-VECTOR) $\begin{bmatrix} - \\ - \\ - \\ - \\ - \end{bmatrix}$

<u>NOTE 1</u>: SUPPRESS n, USUALLY.

<u>NOTE 2</u>: ROW/COLUMN VECTORS HAVE THE SAME DATA PACKAGED DIFFERENTLY.

NOTATION: $\vec{v}$, $\mathbf{v}$, $\underset{\to}{v}$, $\bar{v}$, $\underline{v}$, ...

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

(ROW / COLUMN) VECTORS OF A MATRIX

$A_{m\times n} \to$ ORDERED COLLECTION OF n (COLUMN m-VECTORS)

$$[\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \vec{a}_4] = \quad A = \begin{bmatrix} 2 & 0 & -4 & 9 \\ -3 & -1 & 4 & 1 \\ 8 & -3 & -3 & 2 \end{bmatrix}$$

$$\vec{a}_1 = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$$

$$\vec{a}_2 = \begin{bmatrix} 0 \\ -1 \\ -3 \end{bmatrix} \qquad \vec{a}_3 = \begin{bmatrix} -4 \\ 4 \\ -3 \end{bmatrix} \qquad \vec{a}_4 = \begin{bmatrix} 9 \\ 1 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & -4 & 9 \\ -3 & -1 & 4 & 1 \\ 8 & -3 & -3 & 2 \end{bmatrix}$$

$\vec{b}_1 = [2 \quad 0 \quad -4 \quad 9]$

$\vec{b}_2 = [-3 \quad -1 \quad 4 \quad 1]$

$\vec{b}_3 = [8 \quad -3 \quad -3 \quad 2]$

$\left. \begin{array}{l} m \times n \\ \text{MATRIX} \end{array} \right\} \longrightarrow$ A COLLECTION OF m (ROW n-VECTORS)

$$A = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \\ \vec{b}_3 \end{bmatrix}$$

$$A = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \end{bmatrix}$$

$\vec{a}_j$: COLUMN m-VECTOR

$$A = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \\ \vdots \\ \vec{b}_m \end{bmatrix}$$

$\vec{b}_j$: ROW n-VECTOR

e.g.

If $\mathbf{a}_1 = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 0 \\ -5 \end{bmatrix}$, and $\mathbf{a}_3 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$,

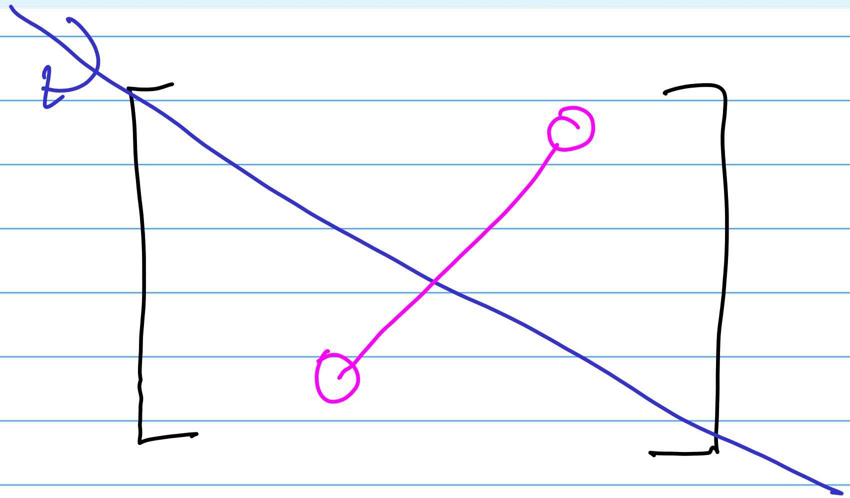
$$\begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -2 \\ -7 & -5 & 4 \end{bmatrix}$$

TRANSPOSE OF A MATRIX

DEFINITION 2.1.8

If we interchange the row vectors and column vectors in an $m \times n$ matrix A, we obtain an $n \times m$ matrix called the **transpose** of A. We denote this matrix by A^T. In index notation, the (i, j)-th element of A^T, denoted a^T_{ij}, is given by

$$a^T_{ij} = a_{ji}.$$



$$m \times n \xrightarrow[\text{TRANSPOSE}]{} n \times m$$

$$\begin{bmatrix} 1 & 2 \\ 5 & 7 \\ -3 & 0 \end{bmatrix} \xrightarrow{\text{TRANSPOSE}} \begin{bmatrix} 1 & 5 & -3 \\ 2 & 7 & 0 \end{bmatrix}$$

$$A = [a_{ij}] \quad \begin{pmatrix} 1 \le i \le m \\ 1 \le j \le n \end{pmatrix}$$

$$A^T = [b_{ij}] = [a_{ij}^T] \quad \begin{pmatrix} 1 \le i \le n \\ 1 \le j \le m \end{pmatrix}$$

$$b_{ij} = a_{ij}^T = a_{ji}$$

2x5 MATRIX

3x3 MATRIX

$$A = \begin{bmatrix} -5 & 3 & 0 & -4 & 1 \\ 8 & -4 & -4 & 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 6 & -2 \\ 0 & -3 & 3 \\ -5 & -1 & 1 \end{bmatrix}$$

$$\underset{5\times 2}{A^T} \quad \begin{bmatrix} -5 & 8 \\ 3 & -4 \\ 0 & -4 \\ -4 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\underset{3\times 3}{B^T} = \begin{bmatrix} 2 & 0 & -5 \\ 6 & -3 & 1 \\ -2 & 3 & 1 \end{bmatrix}$$

ROW VECTORS $\cong$ COLUMN VECTORS

$$\vec{v} \xrightarrow{\text{TRANSPOSE}} \vec{v}^{T}$$

$$\begin{bmatrix} 1 & 3 & 4 \end{bmatrix} \xrightarrow{T} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$(A^T)^T = A$$

SQUARE MATRICES

$m = n$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} = [a_{ij}] \qquad \begin{array}{l} 1 \le i \le n \\ 1 \le j \le n \end{array}$$

DIAGONAL OF A SQUARE MATRIX

$$\text{TRACE} = \operatorname{tr}(A) = \sum_j a_{jj} = a_{11} + a_{22} + \dots + a_{nn}$$

UPPER Δ, LOWER Δ, DIAGONAL

DEFINITION 2.1.10

An $n \times n$ matrix $A = [a_{ij}]$ is said to be **lower triangular** if $a_{ij} = 0$ whenever $i < j$ (zeros everywhere above (i.e., "northeast of") the main diagonal), and it is said to be **upper triangular** if $a_{ij} = 0$ whenever $i > j$ (zeros everywhere below (i.e., "southwest of") the main diagonal). An $n \times n$ matrix $D = [d_{ij}]$ is said to be a **diagonal matrix** if $d_{ij} = 0$ whenever $i \neq j$ (zeros everywhere off the main diagonal).

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 & \cdots \\ 0 & a_{22} & 0 & 0 & \cdots \\ \vdots & 0 & a_{33} & & \vdots \\ & & & \ddots & \\ 0 & \cdots & & 0 & a_{nn} \end{bmatrix}$$

UPPER TRIANGULAR MATRIX

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{n1} \\ & a_{22} & \cdots & \vdots \\ 0 & & \ddots & \vdots \\ & & & a_{nn} \end{bmatrix}$$

LOWER TRIANGULAR MATRIX

$$\begin{bmatrix} a_{11} & & 0 \\ a_{21} & a_{22} & \\ \vdots & \ddots & \\ \vdots & \cdots & a_{nn} \end{bmatrix}$$

UPPER
Δ-LAR

LOWER
Δ-LAR

$$A = \begin{bmatrix} -3 & 3 & 4 \\ 0 & -5 & 1 \\ 0 & 0 & 9 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -5 & 0 & 0 \\ \boxed{0} & 4 & 0 \\ 2 & -2 & -7 \end{bmatrix}$$

$$D = \begin{bmatrix} \boxed{0} & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

DIAGONAL, UPPER Δ-LAR,
LOWER Δ-LAR

DIAGONAL = LOWER Δ-LAR + UPPER Δ-LAR

$$\operatorname{diag}(d_1, \dots, d_n) = \begin{bmatrix} d_1 & & & \text{\huge 0} \\ & d_2 & & \\ & & \ddots & \\ \text{\huge 0} & & & d_n \end{bmatrix}$$

$$\operatorname{diag}(1,2) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \neq \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \operatorname{diag}(2,1)$$

e.g. $\operatorname{diag}(-5, 0, -9, 4)$

4x4

$$= \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -9 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$a_{ij} \neq 0$

ONLY

IF $i = j$ =)

(TO BE CONTINUED!)