

# Homework 1

MATH 201 (Summer 2023, Session A2)

Monday 15<sup>th</sup> May, 2023

## Instructions

- This homework is due on Friday, May 19th at 11 PM Eastern Time.
- Late submissions are not permitted unless there are extenuating circumstances.
- Please read the honesty policy of the course (available on the course webpage) and make sure you understand the collaboration policy.

**Problem 0.** [0 points] Copy paste the following text in the beginning of your submission:

This submission conforms to the honesty policy of the course. In particular, I have not made use of any unauthorized online resources and any collaboration did not violate the expectations outlined in the policy.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

**Problem 1.** [10 points] An urn contains  $n$  blue balls and  $n$  red balls. Two balls are removed from the urn together at random.

- Write down the sample space  $\Omega$ .
- Compute the probability of drawing 2 balls that are different colors.
- Let  $p_n$  be the probability the balls are the same color. Compute  $p_n$  and evaluate

$$\lim_{n \rightarrow \infty} p_n.$$

**Problem 2.** [10 points] Eight rooks are placed randomly on a chess board. What is the probability that none of the rooks can capture any of the other rooks? Chess-less translation: pick 8 unit squares at random from an  $8 \times 8$  grid. What is the probability that no two chosen squares share a row or a column?

**Problem 3.** [10 points] Show that it is not possible to uniformly choose a positive integer at random. (In other words, we cannot define a probability measure on the positive integers that is uniform in the sense that  $P(A) \propto \#A$  for finite  $A \subseteq \mathbb{N}$ .)